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3D wave-packet propagation in inhomogeneous and nonlinear media of Kerr type

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HIGHLIGHTS

- Stable propagation of 3D Gaussian wave packets in nonlinear inhomogeneous fibers.
- New method: complex geometrical optics.
- Spatio-temporal evolution in ordinary differential equations.
- Collapse limit by longitudinal inhomogeneity of the fiber and initial chirps.

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ABSTRACT

Complex geometrical optics (CGO) is applied to spatiotemporal evolution of 3D Gaussian wave packets in nonlinear media of Kerr type. We propose ordinary differential equations (ODEs) for wave packet parameters, which can be derived immediately from the complex eikonal equation and the complex transport equation. The eikonal equation can be used to derive ordinary differential equations for spatial and temporal widths, omitting the complicated variational process used in nonlinear optics. For the combined effect of diffraction, anomalous dispersion and nonlinear refraction we observe two solutions for temporal and spatial widths of the packet propagating in a nonlinear medium of Kerr type: diffraction/dispersion widening and spatiotemporal collapse. Moreover, we discuss the evolution of a 3D Gaussian wave packet in nonlinear inhomogeneous fiber and we present conditions for stable propagation without the collapse effect. We also discuss the influence of initial spatial and temporal chirps on 3D Gaussian wave packet evolution in nonlinear media of Kerr type and in nonlinear inhomogeneous fibers. We demonstrate the ability of CGO method to describe the evolution of a 3D wave packet in a nonlinear dispersive and strongly nonlocal medium, including the existence of a new type of spatiotemporal Gaussian soliton: an accessible spatiotemporal soliton. Moreover, the CGO method is applied to solve a novel problem of the interaction of a pair of 3D Gaussian wave packets in a nonlinear medium of Kerr type.

1. Introduction

Geometrical optics is an asymptotic theory for wave motion based on the assumption that the wavelength of a wave is much smaller than any other characteristic length scale, such as: the radius of curvature of nonplanar wave fronts, diffraction and dispersion scales, inhomogeneity scale, and nonlinear length. Within any form of geometrical optics a beam/pulse or a wave packet propagates along a trajectory called the central ray. The complex geometrical optics (CGO) is the generalization of the geometrical optics in standard Runge–Sommerfeld–Luneburg formulation on the complex domain. The CGO is

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a powerful method because it reduces a four-dimensional spatio-temporal problem for 3D wave packets to a single dimensional one owing to the introduction to the natural paraxial approximation usually applied in the "mathematical apparatus" of geometrical optics. Gaussian beam (GB), pulse or wave packet is localized in the vicinity of propagation axes named as the central ray. There are many other geometrical optics extensions. Two seem to be the most universally recognized; namely nonlinear geometrical optics [1–3] and the method of generalized eikonal approximation [4,5], which come from the Sommerfeld and Runge assumption comprehensively described in *Mathematical Theory of Optics* by Luneburg [6]. Another method, which is able to describe light beam propagation along trajectories is quasi-optics, which deals with the parabolic equation [7–16] supplemented with an equivalent wave optical approach [17]. Nonlinear optics uses also the formalism of theoretical mechanics introducing the variational method [18–20], the method of moments based on the virial theory [21] and approaches which deal with modifications of paraxial wave optics [22] and nonlinear geometrical optics [23]. The commonly accepted numerical approach, used in wave optics to analyze light propagation in nonlinear and graded-index fibers is the beam propagation method (BPM) [24], and the split-step Fourier transform [14].

CGO has two equivalent forms: the ray-based form, which deals with complex rays [25–30], with trajectories in a complex space, and the eikonal-based form, which uses complex eikonal instead of complex rays [31,32]. A surprising feature of CGO is its ability to describe Gaussian beam (GB) diffraction in both ray-based and eikonal-based approaches. Recently, the eikonal-based CGO method has been applied to describe GB evolution in homogeneous [33] and inhomogeneous media [34], nonlinear media of Kerr type [35], nonlinear inhomogeneous fibers [36] and nonlinear saturable media [37].

The propagation of self-trapped optical pulses under the combined effect of dispersion, diffraction and self-focusing is a fascinating topic, which has attracted considerable attention in recent years [38–41]. It is well established that the longitudinal pulse broadening in time due to dispersion can be counterbalanced by nonlinearity-induced self-phase modulation, giving rise to the generation of temporal solitons [40,41]. On the other hand, the transverse broadening of the beam due to diffraction can be counterbalanced by the nonlinearity induced self-focusing leading to the generation of spatial solitons [42–44]. An optical object may also remain self-trapped simultaneously both in space and time under the influence of diffraction, dispersion, self-focusing and self-phase modulation. Such objects have found application in recent nonlinear optics, including digital logic, beam deflection, beam scan and all optical switching. Thus, a simple and effective description of a 3D wave packet in nonlinear media is an important contribution in nonlinear fiber optics including nonlinear graded-index optics.

In this paper we propose an effective approach to develop a standard form complex geometrical optics (CGO). Author's opinion can be recognized to be simpler and more illustrative than the method of generalized eikonal approximation, the variational method and the method of moments described in so far published papers [33–37]. As shown in [45], the eikonal-based CGO method is well able to describe GB evolution in graded-index optical fibers reducing by hundreds the time of numerical calculations compared to the Crank–Nicolson scheme in the Beam Propagation Method (BPM). CGO deals with ordinary differential equations (ODEs) for wave packet spatial and temporal widths and complex amplitude and does not require the complicated variational process used usually in nonlinear optics. This way, CGO method provides simple and effective numerical algorithms for commonly available computer software: e.g. Mathcad, Matlab and Mathematica. CGO method can be useful not only for advanced physicists applying the standard NLS but also for engineers who need a much simpler and illustrative method. The CGO method is applied in this paper for the Gaussian wave packet evolution in nonlinear media of Kerr type with special attention paid to stabilize wave motion and limit spatiotemporal collapse effect. The power of CGO is demonstrated for 3D wave packets evolution in a nonlinear inhomogeneous fiber and in a nonlinear dispersive and strongly nonlocal medium. Moreover, CGO is applied here to solve a novel problem of the interaction of a pair of 3D Gaussian wave packets in a nonlinear medium of Kerr type. In the author's opinion, CGO can be recognized as the simplest method of nonlinear optics, among purely numerical [14–24] and analytical approaches, including the recent extension of self-focusing theory [46–52].

2. Wave Motion of 3D wave packet in a linear dispersive medium using the CGO method

The attempts to reduce the complicated description of wave motion for electromagnetic beams and pulses to solving equations of geometrical optics, i.e. eikonal and transport equations have been made since the late 1960s. The pioneering paper was written by Akhmanov, Sukhorukov and Khokhlov [1,2]. Starting from a nonlinear parabolic equation and using the geometrical optics ansatz given by:

$$u(\mathbf{r}) = \varphi(\mathbf{r}) \exp(ik_0 L(\mathbf{r})), \tag{1}$$

where $\varphi(\mathbf{r})$ is real envelope function and $L(\mathbf{r})$ is real phase function, they obtained a system of coupled partial differential equations for quantities $\varphi(\mathbf{r})$ and $L(\mathbf{r})$, which described self-focusing and diffraction of a spatially limited wave beam propagating in a nonlinear medium of Kerr type. The position vector in Eq. (1) has the form $\mathbf{r} = x\mathbf{e}_x + y\mathbf{e}_y + z\mathbf{e}_z$. Using the Gaussian beam (GB) as a substitution, they obtained an analytical solution for GB width evolution in a nonlinear medium of Kerr type, where three scenarios for the evolution are possible. Namely, defining the parameter of critical power P_{crit}, one obtains three partial cases which deserve to be distinguished separately:

(a) Under-critical case, where total beam power is smaller than the critical: $P < P_{\text{crit}}$. In this case the beam width increases without bound and the beam amplitude tends to zero at $z \to \infty$.