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Simulation of acoustic fields in fluid-, solid- and porous layers by the combined transfer matrix/angular spectrum approach with applications in bioacoustics

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HIGHLIGHTS

- A new method for the calculation of acoustic fields in real experimental setups.
- Accounts for multiple fluid-, linear elastic solid- and/or poroelastic-layers.
- Root mean square error <2% compared to analytical solutions.
- Predicts ultrasonic field in regions where measurements are impossible.
- Applied to ultrasonic bioreactor for experimental design.

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ABSTRACT

A highly accurate semi-analytical method was developed to predict the acoustic field generated by a real transducer in an axisymmetric sonobioreactor consisting of multiple fluid-, linear elastic solid-, and/or poroelastic-layers. The accuracy of the method is independent of the spacing of the grid-points and computational costs are not proportional to the ratio of the system's characteristic dimensions to the acoustic wavelength, both improvements over the use of full numerical methods. Contrary to similar semi-analytical approaches, the method is not limited to the prediction of freely propagating waves. Acoustic reflection and perfect absorption are readily implemented. The method was numerically validated and matched the analytical function describing the pressure amplitude along the axis of a cylindrical transducer with a root-mean-square error of less than 2%. The method was also experimentally validated, but it was shown that the method is not applicable when certain components of the system have a diameter smaller than that of the acoustic beam. The method was used to model an ultrasonic bioreactor as an example problem, where its accuracy and computational efficiency were shown to be instrumental in bioreactor design.

1. Introduction

The use of acoustics in biological and chemical applications is becoming more prevalent, but the mechanisms whereby acoustic waves interact with biological material are not well understood [1]. Experiments in bioacoustics and sonochemistry

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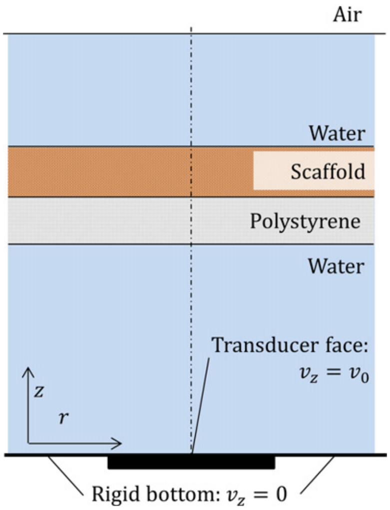

Fig. 1. Diagram of a simplified, axisymmetric ultrasonic bioreactor with boundary conditions. A transducer is placed within an infinite baffle below a polystyrene tissue culture well. The well contains a porous scaffold immersed in water and is acoustically coupled to the transducer by a second water layer. The system is composed of fluids (water and air), solids (polystyrene well bottom) and porous media (scaffold). The simple mathematical model approximates each acoustic layer as infinitely wide (color online).

are extremely delicate and subject to various sources of error [2]. Uncertainties of up to 700% have been reported in certain *in vitro* ultrasonic experiments, but these variations can be eliminated by analyzing the experimental setup [3]. These setups are often quite complex, involving multiple layers of various materials (solids, liquids, porous media, etc.).

Computational cost poses a large problem to the prediction of acoustic fields when the characteristic dimensions of the system are much greater than the acoustic wavelength or when the system is composed of materials such as linear elastic solids and poroelastic media. Analytical methods can only be applied to the simplest situations and fail when a real transducer (especially in the near-field) or multiple acoustic layers are used. The numerical cost of mesh-based methods is very high: ten nodes per wavelength are recommended for most methods [4], with specialized pseudo-spectral methods requiring two nodes per wavelength [5]. Simulating ultrasound with a frequency of 5 MHz in a system with dimensions of approximately 50 mm × 50 mm (assuming a two-dimensional simulation is sufficient) and acoustic properties comparable to that of water requires more than a million nodes. Finally, mesh-based methods fail at accurately simulating perfectly matched boundary layers [6,7].

Under certain circumstances, the problem can be simplified to allow the use of semi-analytical techniques that can achieve greater accuracy with lower computational costs compared to full numerical methods. The angular spectrum approach (ASA) is one such method which decomposes the acoustic field on a reference plane into component waves using a Fourier transform [8–10]. The component plane waves are propagated in the spectral domain and the inverse Fourier transform is applied to determine the acoustic field at discrete points in the system. Researchers often use the ASA with great success to predict the acoustic field generated by transducers of varying design [10–12]. However, the ASA is usually limited to the prediction of free space wave propagation and does not include the reflection and transmission of acoustic waves in layered media [13–15] or only accounts for periodically layered media [16] or first-order reflections [9,17] which is inadequate when investigating resonance effects and standing wave formation in arbitrary systems. When the ASA is used to calculate transmission and reflection, the method of steepest descent is typically applied to evaluate an integral on a complex contour [18]. This requires analytical knowledge of the reflection- and transmission coefficients. Determining the necessary coefficients is tedious, especially when investigating a large number of different systems containing porous media. Most often, the ASA only provides the acoustic field within a specific layer and not throughout the entire system. Thus, there is a need to develop a method with the following properties: (1) predicts standing wave formation through an arbitrary layered system; (2) can be applied to a variety of materials, including poroelastic media; and (3) has high computational efficiency.

This paper describes the combination of the ASA with another well-established method: the transfer matrix method (TMM) [19], to yield the transfer matrix/angular spectrum approach (TM/ASA). The TM/ASA is valid for an axisymmetric system composed of multiple parallel layers. The layers may be composed of inviscid fluids, isotropic linear elastic solids and porous media (described by Biot theory [20]).

The TM/ASA is numerically and experimentally validated. The stationary wave field generated by the continuous application of ultrasound in a bioreactor is investigated as a model problem to test the applicability of the TM/ASA [21] (see Fig. 1). The bioreactor system presents many different challenges including porous media, unwanted resonance effects, etc., and is therefore representative of a wide variety of applications. The TM/ASA is used to identify possible sources of experimental error and determine ways in which these problems can be avoided by rapidly evaluating a number of different designs. Finally, it is demonstrated how the TM/ASA can be used to inform the design of an ultrasonic bioreactor.