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Monte Carlo simulations of Helmholtz scattering from randomly positioned array of scatterers by utilizing coordinate transformations in finite element method

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HIGHLIGHTS

- Helmholtz scattering from randomly positioned scatterers is analyzed by Monte Carlo.
- Form invariance of Maxwell's equations under coordinate transformations is utilized.
- Repeated mesh generation process is eliminated.
- A simple and single mesh is used by inserting transformation media into domain.
- Computation time is reduced considerably.

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ABSTRACT

Electromagnetic scattering from randomly distributed array of scatterers is numerically analyzed by Monte Carlo simulations by utilizing coordinate transformations in the context of finite element method solution of Helmholtz equation. The major goal in proposed approaches is to place transformation media into computational domain by employing the form invariance property of Maxwell's equations under coordinate transformations, and hence avoiding repeated mesh generation process in multiple realizations of the Monte Carlo method. A simple, single and uniform mesh is used, and only the material parameters of the transformation media are changed with respect to the positions of the objects in each realization. In this manner, computational resources are reduced considerably. The proposed approaches are demonstrated and compared with the standard approach via several numerical simulations. Monte Carlo results are presented in terms of some statistical properties (such as mean, standard deviation, probability density functions approximated by histograms) of radar cross section (RCS) and error values.

1. Introduction

Uncertainties are unavoidable in analysis and design of any scientific or engineering phenomenon. Hence, it has been a problem of long-standing interest to identify random events in a quantitative way and to incorporate these events into the analysis and design of real-life applications. Understanding multiple scattering by randomly positioned obstacles is one of such problems, and plays an important role in various fields of science and engineering (such as elasticity, electromagnetics, acoustics, seismology and geophysics). There are various analytical or numerical approaches in the literature for the

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analysis of elastic [1,2], acoustic [3,4] and electromagnetic [5–13] wave interactions among randomly distributed scatterers. Especially the study of electromagnetic wave interactions in such random media is important in various applications of radar remote sensing. Moreover, uncertainties in positions are important in the design of frequency selective surfaces and metamaterial arrays consisting of periodic or aperiodic elements. With the advent of modern computers, Monte Carlo simulation has been widely used to numerically solve such stochastic problems. Monte Carlo is based on repeated sampling of a stochastic problem by means of a given probability distribution; solving the associated wave equation many times for each sample; and taking average over these multiple realizations to identify the wave behavior on the average sense. The results are analyzed by forming them as a *random process* (or *random field* if the domain of the underlying parameter is space instead of time). In performing multiple solutions, finite element or finite difference methods are useful due to their adaptability to arbitrary geometries and material inhomogeneities in volume scattering problems. However, since a sufficient number of realizations have to be performed in Monte Carlo simulations to ensure accuracy, multiple mesh generations and solutions in finite methods become challenging especially in electrically-large problems requiring large number of unknowns.

The major motivation in this study is to propose different methods that are based on the form invariance of Maxwell's equations under coordinate transformations for the purpose of efficient finite element solution of electromagnetic scattering from randomly distributed array of scatterers. The coordinate transformation technique is based on the duality between the geometry and the material parameters. That is, the medium in which the transformation is applied turns into an anisotropic medium, and Maxwell's equations in this medium keep the same mathematical form in order to mimic the desired field behavior dictated by the coordinate transformation. The main mission of this material medium is to create virtual fields that are equivalent to the desired fields. The coordinate transformation technique is indeed an old topic and can be traced back to Bateman at the very beginning of the 20th century [14]. One well-known application of the coordinate transformation is the perfectly matched layer (PML) approach to achieve mesh truncation in finite methods through the use of artificial complex medium that makes the fields decay away from the geometry in order to mimic the far-field behavior [15–18]. The coordinate transformation is named as complex coordinate stretching in PML nomenclature. This technique was also defined in some other applications [19–21]. Obviously, one breakthrough of the coordinate transformation technique is the design of invisibility cloaking approach in 2006 [22]. Just after the cloaking approach, we developed the "reshaping" approach for the first time in the literature as a generalization of the cloaking approach [23]. The reshaping approach is more general because the cloak transforms an object to a *point*, and thus, makes this object invisible; whereas the reshaper medium transforms an object to another object of different shape with reference to an observer. If the new object is defined as point, then the reshaping approach reduces to the cloaking approach. After the cloaking approach, the coordinate transformation technique has evolved as a widely-used approach for systematic design of various optical and electromagnetic devices, and named as *transformation electromagnetics/optics* [24–47]. As obvious from the historical development of the coordinate transformation technique, although the deformation of space is a well-known technique in general, how and for what purposes it will be used can generate novel techniques and interpretations. Our reshaping approach inspired us to use the principles of coordinate transformation to eliminate certain difficulties that arise in the numerical methods, such as in finite element or finite difference methods. In principle, we modify the computational domain and place suitable transformation media to devise simple and efficient computer-aided simulation tools. We applied this idea to multiscale problems and called such transformation materials "software metamaterials" in [40]. In addition, we used this technique for modeling curved geometries that do not conform to a Cartesian grid especially in finite difference methods [41] and for modeling stochastic electromagnetic problems having significant uncertainty, such as rough surface scattering problems [42–46]. In the current paper, our aim is to employ the transformation invariance of Maxwell's equations for the solution of scattering from randomly positioned obstacles to realize multiple realizations in Monte Carlo simulations (corresponding to different positions of the obstacles) in a fast and efficient manner by using a single and simpler mesh and by locating transformation medium within the computational domain. To the contrary of the standard approach where multiple meshes and matrices are constructed for each realization, a single mesh is created and only the material parameters of the transformation medium are modified for each different position. The constitutive parameters of the medium are determined by the Jacobian of the coordinate transformation. In this manner, considerable saving in computational resources is achieved, and the mesh generation process is simplified to a great extent.

The paper is organized as follows: Section 2 presents three approaches to solve and stochastically analyze the scattering problem involving randomly distributed array of scatterers: (i) a transformation medium is designed for each scatterer, (ii) a single transformation medium is designed around all scatterers, and (iii) transformation media are designed through successive translations and compressions. In Section 2.4, a general framework of the computation of the material parameters under a general coordinate transformation is summarized. Each approach is validated through several Monte Carlo simulations based on the finite element solution of Helmholtz equation in electromagnetic scattering problems, and the results are presented in Section 3. It is useful to note at this point that the proposed approaches are demonstrated for the two-dimensional (2D) Helmholtz equation. However, the defined coordinate transformations are also valid in three-dimensional (3D) space. The derivations of the material parameters and the field formulations are also valid in 3D. Hence, the applications can easily be extended to 3D problems in a straightforward manner. It is evident that the computational gains of the proposed approaches in 3D are much greater than what is expected in this study. Finally, Section 4 draws some conclusions.

Throughout the paper, the suppressed time dependence of the form $\exp(j\omega t)$ is assumed.