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On Green's correlation of Stokes' equation

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ABSTRACT

The derivation of Green's correlation naturally arises when identifying a linear propagation medium with uncontrolled random sources or ambient noise. As expected, this involves convolution of the well known Green's function with its time-reversed version. The purpose of this paper is to derive a general expression of Green's correlation function of a linear visco-acoustic propagation medium, in which the pressure field satisfies Stokes' equation. From the expression obtained for a visco-acoustic medium, the Ward identity that was recently obtained for unbounded media is extended to the case of bounded propagation media. This extension appears necessary as the unbounded model is not valid in many practical cases, as for acoustic rooms for example. It is illustrated with both simulations and real-world aerial acoustics experimental data recorded in a closed room and in the framework of passive identification. In these experiments, Green's correlation is estimated by the classical coda-based approach, and the performances are studied in this new context.

1. Introduction

Retrieving the parameters of a linear propagation medium is still an active research topic in many domains, such as wave celerity map estimation in medium tomography, impulse response estimation in wireless communications, and modal pulsation estimation in modal analysis of mechanical structures. Indeed, this list is not exhaustive. When controlled sources can be used, Green's function of the explored propagation medium relates a known excitation source to the generated wave measured by a set of sensors. In this situation, received and emitted signals are processed to estimate Green's function and then to extract the parameters related to the propagation medium and the sensor positions. In a passive context, ambient sources are used (see for example the tutorial [1]). Green's functions cannot be extracted directly, as no deterministic information on the source is available. To circumvent this lack of information, many studies have considered ambient (spatio-temporal) white noise (see [2–7], for a non exhaustive list). For this type of ambient sources, the cross-correlation between two received signals are known as Green's correlation (and also called noise correlation). Green's correlation has a role similar to Green's function, as it relates the auto-correlation of a source to the auto-correlation of the wave generated by this source.

The ambient noise that issues from uncontrolled white sources can be created by thermal noise [2], and by external random excitation [1]. Seismologists have shown a very interesting way to raise the number of 'useful' ambient sources [3].

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They have shown experimentally that the cross-correlation of a coda generated by a propagated impulsion source (such as an earthquake) is a good approximation of white noise cross-correlation (the mathematical proof is still an open problem; see Section 6 of [6]). Briefly, and approximately, this can be explained through ergodic cavity theory. A localized impulse source snared in such a cavity will be 'converted' to a white source during the propagation inside the cavity, as after a certain time (i.e. the mixing time), the cavity 'randomizes' both the directions and the times of arrival. Therefore, the propagated source 'becomes' white after this mixing time, and the origin of the source disappears. This codas-based result is valid in the acoustic framework, and this significantly increases interest in the study of propagation media parameter retrieval from cross-correlations of uncontrolled white sources.

It has been shown that Green's correlation is the space–time convolution of Green's function with its time-reversed version [8,6]. This convolution exists (mathematically) only if the associated propagation equation contains a dissipation term. Many studies have used a constant dissipation model [2,3,9,4,6], which is not realistic in acoustics and elastic propagation media [10,7]. Indeed, in the presence of dissipation, Stokes' equation [11,12] describes the propagation of acoustic waves, and, P-waves and S-waves in the elastic context [11]. Green's function retrieval in the viscous dissipation case was derived in [10] for an unbounded propagation medium. In the same viscous context, the Ward identity (introduced in [8]) that relates Green's correlation of Stokes' equation to Green's function was derived by [10,7]. To the best of our knowledge, Green's correlation for Stokes' equation has never been studied in a general framework (see [5] for Green's correlation derivations in other practical physical contexts).

In this paper, we derive an exact expression of Green's correlation in the time domain. From this expression that is obtained through a modal decomposition, we extend the Ward identity for Stokes' equation obtained in [7] for unbounded propagation media, to any bounded visco-acoustic propagation media. Furthermore, this expression is used to simulate the visco-acoustic Green's correlation, to improve its interpretation, as this is difficult to obtain directly from the equations. Then indoor aerial acoustics experiments with microphones are presented. These illustrate the possibility to retrieve the visco-acoustic Green's correlation with a coda-based approach. The results obtained in these experiments are compared with the theoretical ones developed in the first part of this paper.

This paper is organized as follows:

- Section 2 first introduces Stokes' equation. The associated dispersion relations and modes are computed. Then the visco-acoustic Green's function is derived from a modal decomposition. Codas are presented in the particular regime that combines semi-classical approximation (see part B of [13]) and low attenuation approximation (see Section A of [12]). Finally, Green's correlation of Stokes' equation is introduced and an exact expression is derived in the time domain. From that expression, we establish the Ward identity for Stokes' equation in any bounded media in the presence of low attenuation. Simulations are presented to illustrate the expressions derived.
- Section 3 is dedicated to the experimental Green's correlation retrieval in visco-acoustic propagation media. We introduce the coda-based method, for which performances are established with real data and compared to simulations.
- Section 4 summarizes the results obtained in the previous sections and discusses the identified perspectives in Green's correlation interpretation, and passive parameters retrieval with a coda-based method.

2. Green's correlation of Stokes' equation

2.1. Stokes' equation

Throughout this paper, we consider a three-dimensional (3D) visco-acoustic propagation medium denoted by X and assumed to be *linear* and *homogeneous*. We denote the surface of X by ∂X when this latter is bounded. Bounded and unbounded cases are considered.

We denote the pressure field by $\mathbf{p}$ (fields are in bold). It is a function of time t and position $\underline{x} \in X$ (vectors are underlined): $\mathbf{p}(t, \underline{x})$ is the pressure at time t and position $\underline{x}$. In many applications, dissipation is discarded from the propagation model. However, this is no longer affordable for passive identification purposes [6,8,7]. Based on the results from [7], the viscous dissipation model is considered in the present study. This equation, called Stokes' equation, relates the pressure field generated by an excitation source field $\mathbf{f}$ and writes [11,12]:

$$\frac{\partial^2 \mathbf{p}(t,\underline{x})}{\partial t^2} - \alpha^2 \frac{\partial}{\partial t} \Delta_X \mathbf{p}(t,\underline{x}) - v^2 \Delta_X \mathbf{p}(t,\underline{x}) = \mathbf{f}(t,\underline{x}) \tag{1}$$

where v is the sound speed in the propagation medium, α^2 is the damping coefficient. Δ_X is the Laplace operator defined on X and takes specific values on its boundaries ∂X. To fully described the propagation of the pressure field, Eq. (1) is completed by the initial conditions: causality of the pressure field and of its first derivative with respect to time.

2.2. Spatial modes of Stokes' equation

We introduce the spatial modes of Stokes' equation. Modes formalism will be used to derive exact expressions of the visco-acoustic Green's function and of the visco-acoustic Green's correlation. To derive the modes, we compute the