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Fast and slow dynamics in a nonlinear elastic bar excited by longitudinal vibrations

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HIGHLIGHTS

- Physical model relating ultrasonic experimental observations in complex nonlinear media.
- Sound mathematical properties, combining nonlinear hyperbolic systems and relaxation terms.
- Robust numerical methods.
- Qualitative agreement with experiments.

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ABSTRACT

The dynamics of heterogeneous materials, like rocks and concrete, is complex. It includes such features as nonlinear elasticity, hysteresis, and long-time relaxation. This dynamics is very sensitive to microstructural changes and damage. The goal of this paper is to propose a physical model describing the longitudinal vibrations in heterogeneous material, and to develop a numerical strategy to solve the evolution equations. The theory relies on the coupling of two processes with radically different time scales: a fast process at the frequency of the excitation, governed by nonlinear elasticity and viscoelasticity, and a slow process, governed by the evolution of defects. The evolution equations are written as a nonlinear hyperbolic system with relaxation. A time-domain numerical scheme is developed, based on a splitting strategy. The features observed by numerical simulations show qualitative agreement with the features observed experimentally by Dynamic Acousto-Elastic Testing.

1. Introduction

Understanding the mechanisms of acoustic nonlinearity in heterogeneous materials is an object of intensive studies [1–4]. Experimental evidence has shown that media such as rocks and concrete possess an anomalously strong acoustic nonlinearity, which is of great importance for the description of ultrasonic phenomena including damage diagnostics. Besides the widely-studied nonlinear and hysteretic stress–strain relation [5], a long-time relaxation is also reported by most of the authors [6,7]. This slow dynamics is typically observed in experiments of softening/hardening [8,9], where a bar is forced by a monochromatic excitation on a time interval, before the source is switched-off. During the experiment, the elastic modulus is measured by Dynamic Acousto-Elastic Testing methods. It can be observed that the elastic modulus decreases gradually

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(softening), and then it recovers progressively its initial value after the extinction of the source (hardening). The time scales of each stage is much longer than the time scale of the forcing, which justifies the term "slow dynamics".

The modeling of this slow dynamic effect has been investigated by many authors. An essentially phenomenological model is widely used for this purpose: the Preisach–Mayergoyz model (P–M model) based on the integral action of hysteretic elements connecting stress and strain [10,11,4]. This model initially arose from the theory of magnetism, where the "hysteron" has a clear physical significance. In elasticity, such a physical interpretation is not available. To overcome this limitation and to develop a rigorous theory, various authors have proposed alternative models based on clear mechanical concepts. To our knowledge, the first physical model of slow dynamics was described in [7], where the relaxation was related to the recovery of microscopic contact impeded by a smooth spectrum of energy barriers. This theory was extended in [12,13], and recently improved based on the analysis of inter-grain contacts and the resulting surface force potential with a barrier [4]. Another approach was followed in [14], where the author shows that two rough surfaces interacting via adhesion forces yield dynamics similar to that of the fictitious elements of the Preisach–Mayergoyz space [14].

Here, we present an alternative mechanical description of slow dynamics based on the works of Vakhnenko and coauthors [15,16], where the following scenario is proposed:

- Young's modulus E varies with time. One can write $E(g)$, where g is a time-dependent concentration of defects. It is closely related to the notion of damage in solids mechanics. But contrary to what happens in this irreversible case, where g strictly increases with time, the evolution of g is reversible. Waiting a sufficiently long time, the initial material properties are recovered;
- at equilibrium, stress σ yields a concentration of defects g_σ. The dependence of g_σ with respect to σ is monotonic;
- out of equilibrium, relaxation times are required for g to reach g_σ. Whether $g < g_\sigma$ (increase in the number of defects) or $g > g_\sigma$ (decrease in the number of defects), Vakhnenko et al. state that the time scales differ. The argument is given in Section III of [16]: "there are various ways for an already existing crack in equilibrium to be further expanded when surplus tensile load is applied. However, under compressive load a crack, once formed, has only one spatial way to be annihilated or contracted". In both cases, these relaxation times are much longer than the time scale of the excitation, which explains the slow dynamics.

Comparisons with experimental data are given in Section V of [16], where the authors reproduced experiments done on Berea sandstone [6]. One current weakness is that no micro-mechanical description of the involved defects has been proposed so far. A possible analogy may be found with populations of open/closed cracks filled with air, equivalent to a population of bubbles that relax towards an equilibrium state, depending on the applied stress [17,18]. In counterpart, one attractive feature of Vakhnenko's model is that it combines hyperbolic equations and relaxation terms, which constitutes a sound basis of physical phenomena [19].

The present paper is a contribution to the theoretical analysis of this model and to its practical implementation to describe wave motion in damaged media. First, we point out that no mechanisms prevents the concentration of defects from exceeding 1, which is physically unrealistic. We fix this problem by proposing another expression for the equilibrium concentration. Second, the Stokes model describing viscoelasticity behavior in [16] poorly describes the attenuation in real media, and it is badly suited to time-domain simulations of wave propagation. Instead, we propose a new nonlinear version of the Zener model. This viscoelastic model degenerates correctly towards a pure nonlinear elasticity model when attenuation effects vanish. Moreover, the usual Zener model in the linear regime is recovered [20]. In practice, this model only requires one physical parameter under the assumption of constant quality factor. Third, hyperbolicity is analyzed. Depending on the chosen model of nonlinear elasticity, a real sound speed may be obtained only on a finite interval of strains; this is true in particular with the widely-used Landau's model.

The main effort of Vakhnenko et al. was devoted to the construction of a model of slow dynamics. The resolution of the involved equations was quite rudimentary and not satisfying. Indeed, the equilibrium concentration of defects g_σ was assumed to be known and was imposed (Eq. (17) in [16]), while it depends on σ. But treating the full coupled nonlinear equations is out of reach of a semi-analytical approach, which explains the strategy of these authors. On the contrary, we propose here a numerical method to integrate the full system of equations, involving the nonlinear elasticity, the hysteretic terms of viscoelasticity, and the slow dynamics. Due to the existence of different time scales, a splitting strategy is followed, ensuring the optimal time step for integration. The full system is split into a propagative hyperbolic part (resolved by a standard scheme for conservation laws) and into a relaxed part (resolved exactly).

Our numerical model is very modular. The various bricks (nonlinear elasticity, viscoelasticity, slow dynamics) can be incorporated easily. Numerical tests validate each part separately. When all the whole bricks are put together, typical features of wave motion in damaged media are observed. The softening/hardening experiments are qualitatively reproduced.

2. Physical modeling

In this section, we write the basic components describing the wave motion in a 1D material with damage. The foundations rely on linear elastodynamics, whose equations are recalled in Section 2.1. Then, the soft-ratchet model of Vakhnenko and coauthors is introduced and enhanced in Section 2.2. The fast dynamics is described in Section 2.3, where various known models of nonlinear elasticity are presented, and a nonlinear model of viscoelasticity is proposed. This latter degenerates correctly in the limit cases of linear elasticity or null attenuation.