Wave Motion 51 (2014) 405–417

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

Numerical study of formation of solitary strain waves in a nonlinear elastic layered composite material

Igor V. Andrianov [a], Vladyslav V. Danishevs'kyy [b,*], Oleksandr I. Ryzhkov [b], Dieter Weichert [a]

[a] *Institute of General Mechanics, RWTH Aachen University, Templergraben 64, Aachen D-52062, Germany*

[b] *Department of Structural Mechanics and Strength of Materials, Prydniprovska State Academy of Civil Engineering and Architecture, Chernyshevskogo St. 24-a, Dnipropetrovsk 49600, Ukraine*

ARTICLE INFO

Article history:
Received 15 February 2013
Received in revised form 18 October 2013
Accepted 19 October 2013
Available online 14 November 2013

Keywords:
Composite materials
Nonlinear waves
Solitary waves
Pseudo-spectral method
Fourier–Padé approximants

ABSTRACT

Propagation of nonlinear strain waves through a layered composite material is considered. The governing macroscopic wave equation for the long-wave case was obtained earlier by the higher-order asymptotic homogenization method (Andrianov et al., 2013). Non-stationary dynamic processes are investigated by a pseudo-spectral numerical procedure. The time integration is performed by the Runge–Kutta method; the approximation with respect to the spatial co-ordinate is provided by the Fourier series expansion. The convergence of the Fourier series is substantially improved and the Gibbs–Wilbraham phenomenon is reduced with the help of Padé approximants. As result, we explore how fast and under what conditions the solitary strain waves can be generated from an initial excitation. The numerical and analytical solutions (when the latter can be obtained) are in good agreement.

1. Introduction

Propagation of nonlinear elastic waves in composite materials is accompanied by a number of new phenomena that can never be observed in linear and homogeneous media. Among them, we can point out the dependence of the wave shape, velocity and attenuation upon the amplitude and the formation of solitary bell-shaped strain waves. From the physical standpoint, these specific dynamic properties are caused by the interplay between the effects of nonlinearity and dispersion [1,2].

Nonlinear behaviour of heterogeneous solids can be caused by geometrical, physical and structural factors. Geometrical nonlinearity is described by the Cauchy–Green strain tensor [3]. Physical nonlinearity displays a deviation of the stress–strain relations from the proportional Hooke's law. It can be modelled representing the energy of deformation as a series expansion in powers of invariants of the strain tensor and taking into account the higher-order terms. Such expansion is usually referred to as the Murnaghan elastic potential [4]. However, we should note that the idea of this approach was proposed before by Landau [5]. Originally, the five-constant model of elasticity has appeared much earlier in the work by Voigt [6]. Structural nonlinearity becomes significant if the solid contains highly compressible local defects, such as cracks, voids, dislocations, which is typical for rocks and soils [7–9]. For composite materials, the effect of structural nonlinearity may be also induced by imperfect bonding conditions at the interface between the constitutive components [10]. Nonlinear properties of pre-stressed heterogeneous solids were considered by Abrahams and Parnell [11,12], who studied how the initial finite deformation affects the microstructure and, subsequently, the dynamic response of the material. In the problems

* Corresponding author. Tel.: +380 567652350.
E-mail address: vdanish@ukr.net (V.V. Danishevs'kyy).

0165-2125/$ – see front matter
http://dx.doi.org/10.1016/j.wavemoti.2013.10.003

of wave propagation, nonlinearity leads to the continuous concentration of energy that makes the wave front steeper and steeper.

Dispersion can be qualified as geometrical and structural. Geometrical dispersion does not depend on the presence of microstructure and takes place in homogeneous materials as well. It may appear in beams and plates due to the presence of bending stresses; in rods due to reflections of the wave from lateral surfaces, etc. Meanwhile, structural dispersion is determined essentially by the heterogeneity of composite solids. Successive reflections and refractions of the signal at the interfaces between the components result in the scattering of the wave field. In contrast to nonlinearity, dispersion provides delocalization of energy.

When nonlinearity and dispersion act together, they balance each other's influence. Then, stationary nonlinear waves of a permanent shape can propagate. The increase in nonlinearity leads to the formation of localized solitary waves that travel for long distances keeping the shape and velocity stable.

Study of nonlinear elastic waves has been attracting considerable attention from many authors. Here we are not able to summarize even a small fraction of previous results and, for a detailed review of the subject, refer to the monographs by Erofeyev [13], Porubov [14], and Samsonov [15].

In recent years, propagation of nonlinear waves in heterogeneous materials has been intensively investigated. Different types of higher-order continuum models were used in [13,14,16–21]. The effect of structural dispersion was predicted allowing the elastic medium additional internal degrees of freedom. The corresponding theory can be traced back to the papers by Mindlin [22] and Sun et al. [23]; a connection to Cosserat continua was established by Herrmann and Achenbach [24]. A recent review of the subject was given by Berezovsky et al. [25]. A general theory of heterogeneous media in terms of internal variables is presented in [26].

It should be noted that many approaches described above include a number of phenomenological parameters, which are not known *a priori* and, for real materials, are expected to be determined experimentally.

A different possibility to take into account the influence of the microstructure is provided by the higher-order asymptotic homogenization method (AHM). According to this approach, physical fields in a spatially periodic composite material are represented by two-scale asymptotic expansions in powers of the small parameter $\varepsilon = l/\lambda$, where l is the size of the unit cell and λ is the minimal wavelength. This leads to a decomposition of the final solution into global and local components; the latter can be evaluated successively from a recurrent sequence of cell boundary value problems. Further, application of the volume-integral homogenizing operator allows us to obtain a homogenized constitutive equation that describes the macroscopic behaviour of the composite material. The coefficients of the homogenized equation (so called *effective* moduli) are evaluated based on the information about the properties of the components and the geometry of the microstructure. From the very beginning, the AHM was intended for the determination of quasi-static properties of heterogeneous media and structures [27,28]. Later, taking into account the higher-order terms with respect to ε gave a possibility to predict the effect of structural dispersion and to obtain macroscopic dynamical equations valid for the long-wave case ($l < \lambda$) [29–31].

Modelling elastic waves in layered composites depends on the direction of propagation. When the waves propagate along the laminas, the dispersion of geometrical type is dominant. Porubov et al. [32] proposed an asymptotic model that reduces the initial two-dimensional problem to a single one-dimensional nonlinear governing equation for longitudinal strain waves. The analytical solution for localized solitary waves was obtained. The influence of the delamination was considered by Khusnutdinova and Samsonov [33]. It was shown that splitting of the laminas leads to the fission of an incident nonlinear wave into a sequence of solitary waves of different amplitudes and velocities. The experimental confirmation of this phenomenon was reported by Dreiden et al. [34].

The waves propagating across the lamina undergo the effect of structural dispersion. In this case, the AHM can be effectively used. Linear waves were studied in [30,31,35]. Recently, Andrianov et al. [36] proposed an extension of the AHM to layered composites with geometrical and physical nonlinearities. The homogenized nonlinear wave equation was derived and analytical solutions for stationary strain waves were presented.

Stationary solutions require specific initial conditions (generally, in the self-reproducing form). From the practical point of view, it is often more important to trace the evolution of an arbitrary initial input. Study of non-stationary dynamic processes may help to answer how fast and under what conditions nonlinear waves of a permanent shape can be generated. Analytical solutions of unsteady problems are available only in some partial cases, when the governing nonlinear equations are integrable [37–39]. Otherwise, numerical procedures should be applied.

There are many works devoted to the numerical simulation of nonlinear dynamic processes in homogeneous systems. Formation of solitary waves from a periodic input was considered by Bridges [40], Rednikov et al. [41], and Salupere et al. [42]. Evolution of localized initial pulses was investigated in [14,43,44]. It was shown that depending on the shape of the initial wave profile (harmonic, Gaussian, rectangular, etc.) and on the sign of the perturbation (compression or tension) different scenarios can be realized, including formation of a single solitary wave, generation of a sequence of solitary waves, and dispersion of the initial pulse without formation of any localized solutions. Harmonic inputs were also used to study interactions of the bell-shaped waves, where no periodic wave structure of a permanent shape arises [42,45].

The present paper aims to extend these results to heterogeneous solids. We shall investigate non-stationary nonlinear strain waves in a layered composite material. As the governing model, we use the macroscopic wave equation (1) obtained previously by the higher-order AHM [36]. From the mathematical standpoint, it coincides with the well-known Boussinesq equation.