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Numerical study of nonlinear interaction between a crack and elastic waves under an oblique incidence

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HIGHLIGHTS

- Finite Element Modeling of wave nonlinear interaction with a crack.
- The crack is modeled by an interface of unilateral contact with Coulomb's friction.
- Explanation of the generation of higher harmonics by the study of contact stresses.
- Study of the coupling between normal and tangential behavior.

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ABSTRACT

A Finite Element (FE) model is proposed to study the interaction between in-plane elastic waves and a crack of different orientations. The crack is modeled by an interface of unilateral contact with Coulombs friction. These contact laws are modified to take into account a pre-stress σ_0 that closes the crack. Using the FE model, it is possible to obtain the contact stresses during wave propagation. These contact stresses provide a better understanding of the coupling between the normal and tangential behavior under oblique incidence, and explain the generation of higher harmonics. This new approach is used to analyze the evolution of the higher harmonics obtained as a function of the angle of incidence, and also as a function of the excitation level. The pre-stress condition is a governing parameter that directly changes the nonlinear phenomenon at work at the interface and therefore the harmonic generation. The diffracted fields obtained by the nonlinear and linear models are also compared.

1. Introduction

The evaluation of damage at an early stage of fracture is relevant in many technologies such as nuclear power plants or aeronautics, for example. Ultrasonic methods based on linear wave scattering are efficient for detecting defects and characterizing material elasticity, but are less sensitive to micro-cracks or closed cracks. Using the nonlinear behavior of these defects, nonlinear ultrasonic techniques such as nonlinear resonance [1], sub- and higher-harmonic generation [2], and frequency-modulation [3] have been shown to be sensitive to micro-cracks or closed cracks. For an overview of nonlinear acoustics applications see [4–6]. When an ultrasonic wave with large enough amplitude is incident on a contact with a frictional interface (e.g. closed cracks), higher harmonics appear in the frequency spectrum of transmitted and reflected waves. This effect, called the Contact Acoustic Nonlinearity (CAN) [2], is of increasing interest for the characterization of closed cracks or imperfectly bonded interfaces.

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Measurements of second harmonic generation for normal incidence of longitudinal waves on a contacting interface between aluminum blocks have been reported in [7,8]. These experiments indicate that the amplitude of the second harmonic decreases rapidly with applied normal contact pressure initially, and then falls off in magnitude at a lesser rate. These findings are in agreement with experimental measurements on contacting adhesive bonds [9]. Both longitudinal and shear wave propagation through a rough surface were investigated numerically in [10] using an interface contact model based on Hertz theory, for time harmonic incidence. The partial contact model was subsequently applied to model scattering from surface breaking cracks [11], and numerical simulations indicated the efficient production of second harmonics. Another approach consists in introducing interface stiffness to account for quantitative transmission and reflection wave and harmonic evolution [7,10,12]. Delrue and Van Den Abeele have worked on the nonlinear behavior of clapping occurring at a delamination in a composite plate [13]. They modeled the interface using a spring model in an FE model and showed that sub and higher harmonic generation is located at the delamination and can thus be detected. Müller has studied lateral impacts on beam-type structures, showing that intermittent contacts lead to the generation of higher harmonics. This phenomenon is modeled by Newton's impact law introduced in a Finite Element (FE) model [14]. Using this law, the dissipative effect of the contact behavior can be reproduced. Time domain implementations such as the Boundary Element Method (BEM) have been used to study an SH slip motion on an arbitrary interface [15]. Using a generalization of this method to include in-plane motion, the interaction between a P wave or an SV wave with a crack under normal incidence was studied for a pre-opened crack or a pre-stressed crack [16,17]. P waves correspond to longitudinal waves, whereas SH and SV waves correspond respectively to out-of-plane shear waves and in-plane shear waves. The crack was modeled by a unilateral contact interface with Coulombs friction. It was shown in [16,17] that the amplitudes of the higher harmonics of the scattered far-fields can be useful in determining both the pre-stress and the frictional coefficient. The same model was also used to treat the interaction between a crack and an SV wave for a given angle of incidence [17]. In this case, the crack was initially closed, and was free of any pre-stress. The solution for the far-field was obtained but the frequency content was not analyzed. When applied to a crack, a traction or a shear stress led to stress singularities at the crack tips.

Numerical methods have been specifically developed for addressing the modeling of a crack in elastic and non-elastic solids. The extended (XFEM) [18] and smoothed (SFEM) [19,20] finite element methods are of particular interest in fracture mechanics to compute the stress intensity factors (SIF) or crack propagations. These methods, as well as the meshfree methods [21], have indeed the advantage of enabling moving discontinuities or large strains to be modeled without being dependent on the mesh quality.

The present study considers the nonlinear interaction of an ultrasonic wave with a closed crack of finite extent. The fracture mechanics aspects (SIF, crack propagation) are not of concern here. The crack is pre-stressed and modeled by a unilateral contact with Coulomb's friction. The resolution is done numerically in the time domain with a Finite Elements (FE) model, presented in Section 2. The mesh is refined around the crack tips in order to ensure convergence. In this study, only the wave with the same polarization as the incident wave is considered. For a P wave and an SH wave, it is of interest to obtain the evolution of the higher harmonics as a function of angle of incidence. Indeed, under normal incidence, a longitudinal wave leads to clapping, whereas a shear wave leads to sliding. Under oblique incidence, both phenomena can occur and be coupled, changing the nonlinear response of the system. Results for the evolution of the higher harmonics as a function of level of excitation or angle of incidence are given in Section 3 then discussed in Section 4 when considering the contact stresses in the crack. The contact stresses are related to the kind of nonlinearity in play at the interface and explain the generation of higher harmonics.

2. Finite element model

A Finite Element model is built using the software Plast2 [22] to treat the interaction between an acoustic wave and a crack of finite extent. An isotropic and homogeneous solid is considered; its mechanical properties are those of aluminum. The Young's modulus is $E = 69$ GPa, the Poisson's coefficient is $\nu = 0.33$ and the density is $\rho = 2700\ \mathrm{kg\ m^{-3}}$. Perfectly matched layers (PML) are set on both sides of the solid to model an infinite solid in direction $\boldsymbol{x}$. A plane wave of five cycles is generated on the upper face of the solid with a central frequency of 0.5 MHz for the shear wave and 1 MHz for the longitudinal wave throughout this study. Finally, the solid contains a crack tilted at an angle θ. Fig. 1 shows the characteristics of the model.

The crack tips give rise to stress singularities ($\sigma \sim 1/\sqrt{r}$ where r is the radial distance from the crack tip). Several methods have been developed to take into account stress singularities in an FE model. One possibility consists in using shape functions that have singularity terms of the desired order. New functions can be defined [23], or the supports of these functions can be changed, defining the so-called quarter-point elements [24]. As explained in the introduction, more recent techniques based on the enrichment of the element (XFEM [18], SFEM [19,20]) can be used to easily take into account the stress singularities. These methods have been developed mainly for the fracture mechanics. Here, the computation of the SIF or the crack propagation is not of interest, therefore, the simplest way to deal with the stress singularities is to refine the mesh, in order to ensure the convergence of the model and maintain its accuracy. First, the convergence has been studied for a contact interface between two solids (the interface crosses the full width of the sample), in order to demonstrate that the model is effective for studying the CAN. The decreasing of the error with respect to a reference solution obtained with a very fine mesh enables us to set the maximum size of the elements to $a = 0.2$ mm. Thus, the wavelength corresponding to the third harmonic is divided into at least ten elements ($\lambda_{3f}/a \simeq 10$). After defining the global mesh, the next step concerns the handling of the stress singularities in the case of a crack. The solutions for the displacements generated by a wave obtained