Wave Motion 51 (2014) 466–475

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

Elastic wave scattering and dynamic stress concentrations in exponential graded materials with two elliptic holes

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HIGHLIGHTS

- FGMs used in this paper are developing materials other than traditional materials.
- Solving the elastic wave problem for two scattered bodies becomes complex.
- In especial, the scattered bodies studied are non-circular.
- The complex function method used in elastodynamics is another bright spot.

ARTICLE INFO

Article history:
Received 27 January 2013
Received in revised form 16 October 2013
Accepted 2 November 2013
Available online 26 November 2013

Keywords:
Elastic waves scattering
Dynamic stress concentrations
Conformal mapping methods
Local coordinate system
Graded materials
Two elliptic holes

ABSTRACT

Based on the elastodynamics, employing complex functions and conformal mapping methods, and local coordinates, the scattering of elastic waves and dynamic stress concentrations in infinite exponential graded materials with two holes are investigated. A general solution of the problem and expression satisfying the given boundary conditions are derived. The problem can be reduced to the solution of an infinite system of algebraic equations. As an example, numerical results of dynamic stress concentration factors for two elliptic holes in exponential graded materials are presented, and the influence of incident wave number and holes spacing on dynamic stress distributions is analyzed.

1. Introduction

Elastic wave methods can be used to describe and simulate the stress strain states which are produced by a variety of dynamic loads in solid media or structures. The elastic wave propagation, scattering and dynamic stress concentration, as well as the localization of vibration in structures with cutouts are important frontier problems in the realm of mechanics. The investigations on these problems can promote the innovation and development of classic structural dynamics and their solving methods.

Nowadays functionally graded materials have been widely used in aviation, aerospace, shipping and mechanical engineering. Stress analysis and strength designs of structures are crucial to structure designs. To meet the requirements of engineering designs, it is unavoidable to make holes in functionally graded materials. Holes bring about stress

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0165-2125/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.wavemoti.2013.11.005

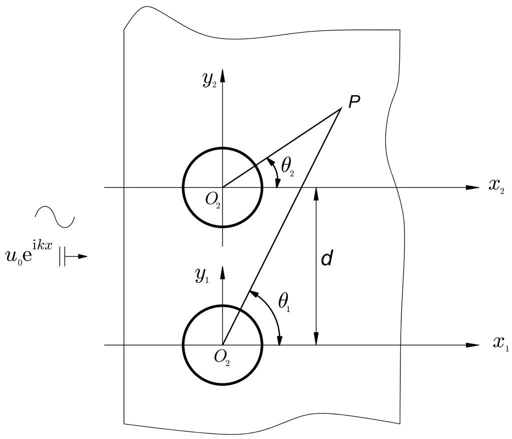

Fig. 1. Schematic of the incident of elastic waves in infinite graded materials.

concentrations, especially dynamic stress concentrations, which will reduce the loading capacity and life time of the structures. Therefore, the study on the scattering of elastic waves and dynamic stress concentrations in graded materials with holes is an important project.

The single circular hole is one of the most simple research models, which Pao and Maw [1,2] made exhaustive discussions. Complex function method proposed by Muskhelishvili [3] is applied to the elastostatics problems in two dimensions with the noncircular holes, which is considered a groundbreaking work. Thereafter, the complex function method developed by Liu et al. [4] is applied to the elastodynamics problems in two dimensions with the arbitrary shape holes. Li et al. [5] worked out at the dynamic stress intensity factor of a cylindrical interface crack in functionally graded materials by means of Fourier transform methods. Numerical methods are widely used in modern elastodynamics researches. Liu et al. [6,7] investigated the transient response of an embedded crack and edge crack perpendicular to the boundary of an orthotropic functionally graded strip. S. Ueda [8] discussed the surface crack problem for a layered plate with a functionally graded non-homogeneous interface. Ma et al. [9] studied the dynamic behavior of a finite crack in functionally graded materials subjected to the normally incident elastic harmonic waves. Fang et al. [10] made a research on the dynamic stress of a circular cavity buried in a semi-infinite functionally graded material subjected to shear waves. C.H. Daros [11] used the boundary element method to study SH-waves in a class of inhomogeneous anisotropic media and presented numerical results. Recently, P.A. Martin [12] investigated the scattering by defects in an exponentially graded layer and Q. Yang et al. [13,14] discussed stress analysis of a functional graded material plate with a circular hole.

Based on the theory of elastic wave [15,16], we discuss the scattering of elastic waves and dynamic stress concentrations by two holes in exponential graded materials. By using the complex function method and local coordinate system, the solutions of this problem under the influence of incident waves with different wave numbers are obtained. Then a solution of the problem can be reduced to the solution of the infinite system of algebraic equations. At last, the numerical results of dynamic stress concentration factors around two stress-free elliptic holes in exponential graded materials are presented and discussed.

2. Equation of wave motion and its solution

Consider the infinite exponential graded materials in which the shear modulus and density change continuously along with the x direction with the variation form as

$$\mu(x) = \mu_0 \exp(2\beta x), \qquad \rho(x) = \rho_0 \exp(2\beta x) \tag{1}$$

where μ_0, ρ_0 are the shear modulus and density at $x = 0$, and β is the non-homogeneous coefficient which stands for the spatial variation of the shear modulus and density in exponential graded materials.

The incidence of anti-plate shear waves along the x direction is considered in infinite exponential graded materials with two holes depicted in Fig. 1. We can express the governing equation of elastic wave motions in exponential graded materials as

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = \rho(x)\frac{\partial^2 u}{\partial t^2} \tag{2}$$

in which τ_{xz}, τ_{yz} are the shear stress.

We can write the constitutive relation for the anti-plane shear deformation

$$\tau_{xz} = \mu(x)\frac{\partial u}{\partial x}, \qquad \tau_{yz} = \mu(x)\frac{\partial u}{\partial y}. \tag{3}$$