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Breather-like director reorientations in a nematic liquid crystal with nonlocal nonlinearity

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HIGHLIGHTS

- The nature of nonlinear molecular deformations in NLC systems has been investigated.
- We obtain the nonlocality induced breather-like properties of director fluctuations.
- These characteristics are used for novel all-optical and switching devices based on NLC systems.

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ABSTRACT

The nature of nonlinear molecular deformations in a homeotropically aligned nematic liquid crystal (NLC) is presented. We start from the basic dynamical equation for the director axis of a NLC with elastic deformations and adopt space curve mapping procedure to analyze the dynamics. The NLC is governed by an integro-differential perturbed nonlocal nonlinear Schrödinger equation and we solve the same using Jacobi elliptic function method aided with symbolic computation and construct an exact solitary wave solution. In order to better understand the effect of nonlocality on the director reorientations of nematic liquid crystal, we have constructed the component forms of director axis using Darboux vector transformation. This intriguing property as a result of the relation between the coherence of the breather-like solitary deformation and the nonlocality reveals a strong need for a deeper understanding in the theory of self-localization in NLC systems.

1. Introduction

Nonlinear dynamics of liquid crystals has been a subject of intensive study for more than two decades [1,2]. Not surprisingly, in both basic and applied research solitons have been found to have important effects in the mechanical, hydrodynamical and thermal properties of highly nonlinear liquid crystals and play an important role in the switching mechanism of some ferroelectric liquid crystal displays [3,4]. In a nematic liquid crystal (NLC), the molecules are considered as elongated rods which are positionally disordered but reveal a long-range orientational order. This property is described on a mesoscopic level by a unit vector $\mathbf{n}(\mathbf{r})$, which is called the director axis pointing in the direction of the average molecular alignment. Due to the absence of a permanent polarization in the nematic phase the director just indicates the orientation but it has neither head nor tail. However, the director reorientation or molecular excitation in NLC systems takes place due to elastic deformations such as splay, twist and bend [5].

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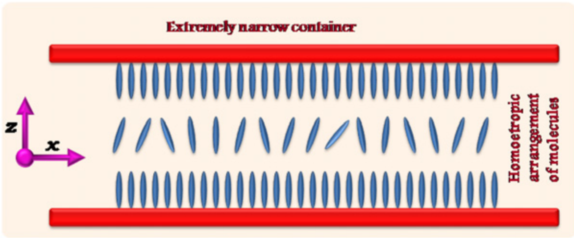

Fig. 1. A sketch of the quasi one-dimensional nematic liquid crystal system contained in an extremely narrow infinite container.

The nonlinearity due to reorientation effect in a nematic phase leads to numerous effects not observed in any other types of material medium. Mostly the nonlinear effects are based on molecular reorientation, and this behavior leads to soliton and under suitable conditions solitary waves can exist in NLC systems which has been investigated extensively both from theoretical and experimental point of view [6–10]. Propagation of solitons in an uniform shearing nematics was first studied by Lin et al. [5] and Zhu experimentally confirmed the existence of solitary like director wave excited by a mechanical method [6]. Magnetically induced solitary waves were found to evolve in a NLC which was first discovered by Helfrich [7] and later confirmed by Leger [8]. Further Migler and Meyer reported the novel nonlinear dissipative dynamic patterns and observed several types of soliton structures in the NLC systems under the influence of a continuously rotating magnetic field [9]. More recently Daniel et al., studied the director dynamics in a quasi-one-dimensional NLC under elastic deformations in the absence of an external field without imposing the one constant approximation. The molecular deformation in terms of a rotational director axis field is found to exhibit localized behavior in the form of pulse, hole and shock as well as solitons [11,12].

In the present paper, we assume that our liquid crystal system is contained in an extremely narrow container with homeotropic alignment of molecules with a strong surface anchoring at the boundaries as illustrated in Fig. 1. In this case, we assume that the molecular field due to elastic energy is not parallel with the director axis which necessarily involves splay and bend type deformations in addition to twist. We make an attempt to demonstrate the existence of the breather-like director dynamics by employing the Jacobi elliptic function method to solve the associated dynamical equation and reveal the underlying nonlinear dynamics.

The plan of the paper is as follows. We construct the dynamical torque equation representing the director dynamics and map the same to an equivalent perturbed nonlocal nonlinear Schrödinger (NLS) equation using space-curve mapping procedure in Section 2. In Section 3, we solve the integro-differential perturbed nonlocal NLS equation by means of symbolic computation and Jacobi elliptic function method is employed to construct an exact solitary wave solution. In Section 4, we construct the components of the director axis using the Darboux vector transformation. Finally we conclude our results in Section 5.

2. Frank elastic theory model of NLC

Liquid crystals are anisotropic materials with an anisotropy axis along the molecular orientation. At a given temperature, NLC molecules fluctuate around the mean direction defined by the director $\mathbf{n}(\mathbf{r})$. The distortion of the molecular alignment corresponds to the free energy density of NLC [13] is given by

$$f = \frac{1}{2}\left\{K_1(\nabla \cdot \mathbf{n})^2 + K_2(\mathbf{n} \cdot \nabla \times \mathbf{n})^2 + K_3(\mathbf{n} \times \nabla \times \mathbf{n})^2\right\}, \tag{1}$$

where K_i represents the elastic constants for the three different basic canonical deformations, splay ($i = 1$), twist ($i = 2$) and bend ($i = 3$). These constants are phenomenological parameters which can be connected with the intermolecular interaction giving rise to the nematic phase. Usually $K_3 > K_1 > K_2$, but Eq. (1) is simplified by assuming the one-elastic constant approximation $K_3 \simeq K_1 \simeq K_2 = K$. We ignore the spatial variations in the degree of orientational order and describe the NLC in terms of the director rather than the order parameter tensor. We also ignore the effects of flow and work in the one-elastic approximation. Under this approximation, the free energy density given in Eq. (1) takes the simple form

$$f = \frac{K}{2}\{(\nabla \cdot \mathbf{n})^2 + (\nabla \times \mathbf{n})^2\}. \tag{2}$$

To obtain the equation of motion, it is necessary to describe the generalized thermodynamic force acting on the director. We note that the molecular field $\mathbf{h}_{el}$ corresponding to the pure elastic deformations using the Lagrange equation $h_i = -\frac{\partial f}{\partial \mathbf{n}_i} + \partial_j \frac{\partial f}{\partial \mathbf{g}_{i,j}}$, $i, j = x, y, z$ and $\mathbf{g}_{ij} = \partial_j \mathbf{n}_i$ satisfies $\tilde{\mathbf{h}} = \mathbf{h} - (\mathbf{h} \cdot \mathbf{n})\mathbf{n}$ introduced by de Gennes [1]. The quantity $(\mathbf{h} \cdot \mathbf{n})$ may be interpreted as the Lagrange multiplier associated with the constraint $\mathbf{n}^2 = 1$ and the condition for equilibrium is that $\tilde{\mathbf{h}} = 0$ or $\mathbf{h} = (\mathbf{h} \cdot \mathbf{n})\mathbf{n}$. Nematic liquid crystals are charge carrying fluids with long range, uniaxial orientation and molecular alignment giving rise to anisotropic, macroscopic properties. By virtue of the anisotropic properties of nematic liquid crystals, it is advantageous to study the dynamics of director axis $\mathbf{n}(\mathbf{r})$ instead of studying the dynamics of all the