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Rayleigh waves in an isotropic elastic half-space coated by a thin isotropic elastic layer with smooth contact

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HIGHLIGHTS

- The propagation of Rayleigh waves in an elastic half-space coated by a thin elastic layer is considered.
- The half-space and the layer are both isotropic and the contact between them is smooth.
- By using the effective boundary condition method an approximate secular equation of fourth-order has been derived.
- From it, an explicit third-order approximate formula for the Rayleigh wave velocity has been established.
- The approximate secular equation and the formula for the velocity will be useful in practical applications.

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ABSTRACT

In the present paper, we are interested in the propagation of Rayleigh waves in an isotropic elastic half-space coated with a thin isotropic elastic layer. The contact between the layer and the half space is assumed to be smooth. The main purpose of the paper is to establish an approximate secular equation of the wave. By using the effective boundary condition method, an approximate, yet highly accurate secular equation of fourth-order in terms of the dimensionless thickness of the layer is derived. From the secular equation obtained, an approximate formula of third-order for the velocity of Rayleigh waves is established. The approximate secular equation and the formula for the velocity obtained in this paper are potentially useful in many practical applications.

1. Introduction

The structures of a thin film attached to solids, modeled as half-spaces coated with a thin layer, are widely applied in modern technology. Measurement of mechanical properties of thin supported films is therefore very significant [1]. Among various measurement methods, the surface/guided wave method [2] is used most extensively in which the Rayleigh wave is a most convenient tool. For the Rayleigh-wave approach, the explicit dispersion relations of Rayleigh waves supported by thin-film/substrate interactions are employed as theoretical bases for extracting the mechanical properties of the thin films from experimental data. They are therefore the key factor of the investigations of Rayleigh waves propagating in half-spaces covered by a thin layer. Taking the assumption of a thin layer, explicit secular equations can be derived by replacing approximately the entire effect of the thin layer on the half-space by *the so-called effective boundary conditions which relate the displacements with the stresses of the half-space at its surface*. For obtaining the effective boundary conditions Achenbach [3] and Tiersten [4] replaced the thin layer by a plate modeled by different theories: Mindlin's plate theory and the plate theory of low-frequency extension and flexure (classical plate theory), while Bovik [5] expanded the stresses at the top surface of

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the layer into Taylor series in its thickness. The Taylor expansion approach was then employed by Niklasson [6], Rokhlin [7,8], Benveniste [9], Steigmann and Ogden [10], Steigmann [11], Ting [12], Vinh and Linh [13,14], Kaplunov and Prikazchikov [15] to establish the effective boundary conditions.

Achenbach [3], Tiersten [4], Bovik [5], Tuan [16] assumed that the layer and the substrate are both isotropic and derived approximate secular equations of second-order (these equations do not coincide totally with each other). In [10] Steigmann and Ogden considered a transversely isotropic layer with residual stress overlying an isotropic half-space and the authors obtained an approximate second-order dispersion relation. In [17] Wang et al. considered an isotropic half-space covered by a thin electrode layer and the authors obtained an approximate secular equation of first-order. In [13] the layer and the half-space were both assumed to be orthotropic and an approximate secular equation of third-order was obtained. In [14] the layer and the half-space were both subjected to homogeneous pre-strains and an approximate secular equation of third-order was established which is valid for any pre-strain and for a general strain energy function.

In all investigations mentioned above, the contact between the layer and the half-space is assumed to be welded. For the case of smooth contact, there exists only one approximate secular equation of third-order in the literature established by Achenbach and Keshava [3]. This approximate secular equation includes the shear coefficient, originating from Mindlin's plate theory [18], whose usage should be avoided as noted by Muller and Touratier [19], Touratier [20]. This remark was also mentioned in [21].

It should be noted that for the case of smooth contact, one could not arrive at the effective boundary conditions from the relations between the displacements and the stresses at the bottom surface of the layer which were derived by Tiersten [4] and Bovik [5]. In contrast, for the case of welded contact, the effective boundary conditions were immediately obtained.

The main purpose of the paper is to establish an approximate secular equation of Rayleigh waves propagating in an isotropic elastic half-space coated with a thin isotropic elastic layer for the case of smooth contact. By using the effective boundary condition method, an approximate effective boundary condition of fourth-order which relates the normal displacement with the normal stress at the surface of the half space is derived. Using this condition along with the vanishing of the shear stress at the surface of the half-space, an approximate secular equation of fourth-order in terms of the dimensionless thickness of the layer is derived. We will show that the approximate secular equation obtained is a very good approximation. Based on it, an approximate formula of third-order for the velocity of Rayleigh waves is established.

2. Effective boundary condition of fourth-order

Consider an elastic half-space $x_3 \geq 0$ coated by a thin elastic layer $-h \leq x_3 \leq 0$. Both the layer and half-space are homogeneous, isotropic and linearly elastic. The layer is assumed to be thin and has a smooth contact with the half-space. In particular, the normal component of the particle displacement vector and the normal component of the stress tensor are continuous, while the shearing stress vanishes across the interface $x_3 = 0$, see Achenbach [3] and Murty [22]. Note that the same quantities related to the half-space and the layer have the same symbol but are systematically distinguished by a bar if pertaining to the layer.

If it is assumed that a state of plane strain exists, whereby the x_2 component of displacement vanishes and the x_1 and x_3 components are functions of x_1, x_3 and t only, i.e.

$$u_i = u_i(x_1, x_3, t), \qquad \bar{u}_i = \bar{u}_i(x_1, x_3, t), \quad i = 1, 3, \qquad u_2 = \bar{u}_2 \equiv 0 \tag{1}$$

where t is the time. Since the layer is made of isotropic elastic materials, the strain–stress relations take the form

$$\begin{aligned}
\bar{\sigma}_{11} &= (\bar{\lambda} + 2\bar{\mu})\bar{u}_{1,1} + \bar{\lambda}\bar{u}_{3,3}, \\
\bar{\sigma}_{33} &= \bar{\lambda}\bar{u}_{1,1} + (\bar{\lambda} + 2\bar{\mu})\bar{u}_{3,3}, \\
\bar{\sigma}_{13} &= \bar{\mu}(\bar{u}_{1,3} + \bar{u}_{3,1})
\end{aligned} \tag{2}$$

where $\bar{\sigma}_{ij}$ is the stress of the layer, commas indicate differentiation with respect to spatial variables x_k, $\bar{\lambda}$ and $\bar{\mu}$ are Lame constants. In the absent of body forces, the equations of motion for the layer is

$$\begin{aligned}
\bar{\sigma}_{11,1} + \bar{\sigma}_{13,3} &= \bar{\rho}\ddot{\bar{u}}_1, \\
\bar{\sigma}_{13,1} + \bar{\sigma}_{33,3} &= \bar{\rho}\ddot{\bar{u}}_3
\end{aligned} \tag{3}$$

where a dot signifies differentiation with respect to t. From Eqs. (2), (3) we have

$$\begin{bmatrix} \bar{U}' \\ \bar{T}' \end{bmatrix} = \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} \begin{bmatrix} \bar{U} \\ \bar{T} \end{bmatrix} \tag{4}$$

where

$$\bar{U} = \begin{bmatrix} \bar{u}_1 & \bar{u}_3 \end{bmatrix}^T, \qquad \bar{T} = \begin{bmatrix} \bar{\sigma}_{13} & \bar{\sigma}_{33} \end{bmatrix}^T$$