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Eigenfrequency correction of Bloch–Floquet waves in a thin periodic bi-material strip with cracks lying on perfect and imperfect interfaces

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ABSTRACT

We analyse an asymptotic low-dimensional model of anti-plane shear in a thin bi-material strip containing a periodic array of interfacial cracks. Both ideal and non-ideal interfaces are considered. We find that the previously derived asymptotic models display a degree of inaccuracy in predicting standing wave eigenfrequencies and suggest an improvement to the asymptotic model to address this discrepancy. Computations demonstrate that the correction to the standing wave eigenfrequencies greatly improve the accuracy of the low-dimensional model.

1. Introduction

In this paper we present a method to correct discrepancies which arise in the asymptotic approximation of standing wave eigenfrequencies in a thin waveguide containing cracks and different types of interface. Substantial interest in the analysis of waves interacting with waveguide boundaries can be found in the literature. In acoustics and water waves, problems in periodic waveguides have been studied in [1–3], among others. Bi-material structures are widely used across many engineering disciplines, ranging from film coatings to armour production. In such applications, it becomes vital to understand how waves propagate through such structures and to estimate the stresses near the crack tip which may be sufficient to cause defects to propagate, which in turn may cause failure of the entire structure [4].

The effect that interfacial cracks and other defects have upon the behaviour of structures is a particularly active research area attracting significant attention. Modelling of interfacial cracks was studied in the important early papers [5,6]. The study of different types of interfaces, including ideal and non-ideal, is also widely covered in the literature, for example in [7–13], among others. Recently the interaction of an interfacial crack with small impurities has been considered in the asymptotic regime in [14].

The present paper builds upon the results of [15,16] and makes a breakthrough in improving the results in a wide range of cases that are important for applications. The former manuscript uses a weight function approach to construct an asymptotic model for out-of-plane Bloch–Floquet wave propagation in a thin bi-material strip with an array of cracks positioned along the join. The latter considers a similar geometry, but with non-ideal interfaces lying between the cracks. Both papers use a weight function approach to obtain constants describing stress distribution near the crack tips and also derive junction conditions for an asymptotic low-dimensional model. The conditions enable dispersion diagrams to be constructed. Comparison with finite element simulations demonstrates that in both the perfect and imperfect interface cases, the low dimensional model has high accuracy in the cases of the waves that propagate through the strip, usually of

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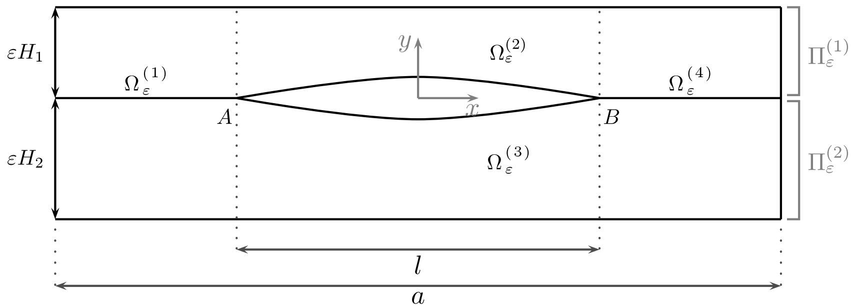

Fig. 1. Geometry of the elementary cell.

the order of 10^{-4}%, but display a larger discrepancy in the case of the standing waves. The size of this discrepancy depends greatly upon material and geometrical parameters but is typically somewhere in the region of 3%–15%. It is this standing wave discrepancy which we aim to address in this paper, by considering the next asymptotic terms for the solution and eigenfrequencies.

The structure of the present paper is as follows. In Section 2 we present the problem formulation before developing the low dimensional model in Section 3. The eigenvalue correction term is derived in Section 4, firstly for an elementary symmetric and homogeneous special case which is easily traceable and enables us to derive the correction term for the standing wave analytically, before considering the fully general case. We then present numerical results which demonstrate the effectiveness of the correction method in a variety of cases and comment on situations where the model fails to give useful information.

2. Problem formulation

The full problem formulation is as given in [16], but is summarised here. The geometry considered is a bi-material strip composed of two materials of shear moduli μ_1 and μ_2, with respective thicknesses εH_1 and εH_2, where ε is a small dimensionless parameter. The elementary cell of the periodic structure is shown in Fig. 1. Along the interface lies a periodic array of cracks of length l, between which the interfaces are imperfect and whose extent of imperfection is described by the parameter τ. The problem is singularly perturbed and so the case $\tau = 0$ which corresponds to the perfect interface case requires different analysis [15] to the imperfect case [16]. The distance between adjacent cracks is $a - l$, and we assume that a, l and H_j are all of the same order. The functions $u^{(j)}(x, y),\ j = 1, 2$, are respectively defined above and below the interface as solutions of the Helmholtz equations

$$\Delta u^{(j)}(x, y) + \frac{\omega^2}{c_j^2} u^{(j)}(x, y) = 0, \quad (x, y) \in \Pi_\varepsilon^{(j)}, \tag{1}$$

where (as indicated in Fig. 1),

$$\Pi_\varepsilon^{(j)} = \left\{ (x, y) \in \mathbb{R} : x \in \left(-\frac{a}{2}, \frac{a}{2} \right),\ (-1)^{j+1} y \in (0, \varepsilon H_j) \right\}. \tag{2}$$

We further assume that $\omega \ll c_j/\varepsilon$; we do not consider high frequency solutions in this paper. High frequency treatments are available in [17] and high frequency long wavelength analysis of hard and soft interfaces can be found in [18]. A zero stress component is imposed in the out-of-plane direction along the top and bottom of the strip, as well as along the face of the crack itself:

$$\sigma_{yz}^{(1)}(x, \varepsilon H_1) = 0, \qquad \sigma_{yz}^{(2)}(x, -\varepsilon H_2) = 0, \quad x \in (-a/2, a/2), \tag{3}$$

$$\sigma_{yz}^{(1)}(x, 0^+) = 0, \qquad \sigma_{yz}^{(2)}(x, 0^-) = 0, \quad x \in (-l/2, l/2). \tag{4}$$

Here, the components of stress in the out-of-plane direction are given by

$$\sigma_{yz}^{(j)}(x, y) := \mu_j \frac{\partial u^{(j)}}{\partial y}, \quad j = 1, 2. \tag{5}$$

Outside the crack, along the boundary between $\Pi_\varepsilon^{(1)}$ and $\Pi_\varepsilon^{(2)}$, the interface is described by the condition

$$u^{(1)}(x, 0^+) - u^{(2)}(x, 0^-) = \varepsilon \tau \sigma_{yz}^{(1)}(x, 0^+), \quad x \in (-a/2, -l/2) \cup (l/2, a/2). \tag{6}$$

Note again that the case $\tau = 0$ corresponds to a perfect interface, while $\tau > 0$ represents imperfect interfaces. We also assume continuity of tractions across the interface

$$\sigma_{yz}^{(1)}(x, 0^+) = \sigma_{yz}^{(2)}(x, 0^-), \quad x \in (-a/2, -l/2) \cup (l/2, a/2). \tag{7}$$