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Analytical study on wave making in a deep wave flume in step-type

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ABSTRACT

Based on the eigenfunction expansion technique, an analytical study on wave generation by a piston wave maker in a deep wave flume in step-type is carried out. By matching 'wavemaker' solutions for the two regions at the cut above the step, the corresponding theoretical velocity potential and wave elevation in the flume are developed. The analytical solution is verified by a higher-order boundary element method (HOBEM). Numerical experiments are carried out to study the effects on the generated wave amplitude in deep water due to the step, the water depths at the up- and down-steps, the distance between the wave maker and the step cut, and the motion period of the wave maker. According to the proposed theory, the suitable water depths, and the wave maker position can be derived to reach the required wave height. Meanwhile, the relationship between the motion amplitude of the wave maker and the expected wave period & amplitude are also presented.

1. Introduction

As deep-ocean resource exploitation has become the focus of the world energy development, the problem of modeling deep-water waves is increasingly important for the design of deep-sea engineering, environmental assessment, and offshore operation safety. There have been many investigations of deep-water wave problems. For example, Newman [1] employed the horizontal step theory to study wave propagation over an infinite step, with constant finite depth on one side of the step and infinite depth on the other. Fenton [2] derived the analytical deep-water wave velocity and elevation by the fifth-order. Ferront [3] used a couple of frequency and time approach to solve the non-linear wave radiation in deep water. Lee et al. [4] considered wave loads of moving objects in deep water by intercepting the domain of finite length. Ning et al. [5] adopted the similar technique to solve deep-water wave problems using an accelerated desingularized BEM. Fonseca and Soares [6] adopted the time-domain Green function satisfying 3D free surface conditions to investigate problems on ships motion with large amplitude in infinite water depth. Ning et al. [7] made the numerical simulation of non-linear regular and focused waves in infinite water depth using source generation technique. However, the application range of the theoretical analysis and numerical simulation is still limited. It is more important to make physical experiments on deep-water wave problems. Due to the expensive cost and great difficulty in building deep wave flumes, it is of great significance for the flume designing to develop exact wave maker theories.

As for the deep-water waves, the particle velocity decreases exponentially with the increasing of water depth. Thus we can build the deep-water flume in step-type and waves are generated through setting the wave maker in a shallow water depth. For the rocker-flap wave maker, we can set it at the step cut and the existing theory [8] is still applicable, while the piston wave maker should be set at the up step. Because of the existence of the wave maker, waves reflected from the step can reach the wave maker and be re-reflected. The waves in the up-step region are different from the incident waves

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in the flat-bottomed flume. Therefore, it is necessary to develop new theories to study wave generation in such deep wave flume.

In this paper, an exact analytical solution is developed for the problem of wave generation by a piston wave maker in a deep wave flume in step-type. Our study is based upon the usual assumptions of linearized water-wave theory and is restricted to the two-dimensional motion associated with wave crests parallel to the step. The fluid is assumed to be ideal, the motion irrotational, and the wave amplitude small compared with the wavelength and the fluid depth. The computational domain in the flume is divided into two regions along the step cut. Consequently, an analytical method of eigenfunction expansion is adopted to calculate the velocity potentials in the up- and down-step regions. Moreover, numerical analysis has been carried out in detail to study the effects on the generated wave height due to the step, the ratio of water depths at the up- and down-steps, the distance between the wave maker and the step cut, and the motion period of the wave maker. It is shown that the wave heights are evidently different from those generated in the flat-bottomed flume. It is also found that both wave heights in the up- and down-step regions vary harmonically with the distance between the wave maker and the step cut, and the oscillating frequency decreases with the increasing of the motion period.

2. Governing equation and boundary conditions

A deep wave flume in step-type is considered here, as shown in Fig. 1. A Cartesian coordinate system is defined with the origin in the plane of the undisturbed free surface $z = 0$, with the z-axis positive upwards, and the x-axis positive rightwards. At the left end of the wave flume, a monochromatic wave is generated by a piston wave maker undergoing the following motion:

$$x(t) = x_0 + \mathrm{Re}[ae^{-i\omega t}] \tag{1}$$

$$U(t) = \mathrm{Re}[-i\omega ae^{-i\omega t}] \tag{2}$$

where $x(t)$ and $U(t)$ are the displacement and the velocity of the wave maker respectively, a and ω are the motion amplitude and the angular frequency of the wave maker, $x_0 = -B$ is the initial position of the wave maker, and $i = \sqrt{-1}$.

The fluid is assumed to be inviscid and incompressible, and the motion is irrotational. Under the further assumption that the motion is simply harmonic in time with the angular frequency ω, the time factor can be separated out and the velocity potential $\Phi(x, z, t)$ can be rewritten as follows:

$$\Phi(x,z,t) = \mathrm{Re}[-i\omega a\phi(x,z)e^{-i\omega t}] \tag{3}$$

where $\phi(x, z)$ is the complex spatial velocity potential with unit amplitude motion of wave maker.

To solve the above problem analytically, the fluid domain can be divided into two regions as depicted in Fig. 1.

1. Region Ω_1

   $-d_1 \leqslant z \leqslant 0, \quad x_0 \leqslant x \leqslant 0$

2. Region Ω_2

   $-d_2 \leqslant z \leqslant 0, \quad 0 \leqslant x \leqslant \infty$

The boundary-value problems describing the two regions can be written as follows:

1. Region Ω_1

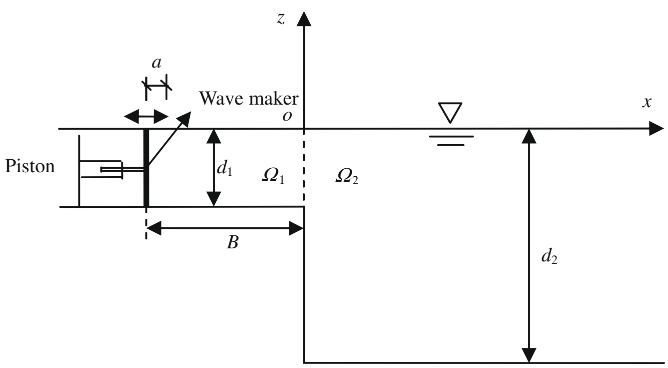

Fig. 1. Definition sketch.