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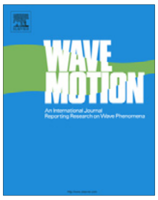

The uses of truncated series in describing the propagation of high-intensity broadband noise

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ABSTRACT

The behavior of the high-frequency part of the spectrum of a broadband noise signal propagating through the atmosphere cannot, according to observation, be accounted for solely by linear mechanisms. Truncated series expansions of the initial signal can afford information on how these linear mechanisms, such as atmospheric attenuation and spherical spreading, might interact with weak nonlinearity to produce the effects observed in the evolved spectrum. Through a Taylor expansion we obtain expressions relating the evolved power spectral density solely to the initial power spectral density and other quantities available directly from measurement, regardless of initial shape. The energy cascade characteristic of broadband noise propagation can be explicitly linked to convolution terms in these expressions. It is also shown that attenuation and spherical spreading affect linear and nonlinear terms in the evolution equation in the same way, producing the same rate of decay. Numerical simulation confirms the validity of these results for distances before shock formation. In the absence of attenuation we find that the range of validity of Taylor expansions is greatly increased by construction of Padé approximants, yielding extremely accurate predictions of the evolved waveforms for distances only slightly shorter than the expected shock formation distance.

1. Introduction

It is normally assumed that linear acoustics mechanisms are sufficient to give a comprehensive description of the propagation of broadband jet noise. However, there are indications that the inverse square law of linear acoustics, coupled with small-signal attenuation along rays, does not adequately describe the evolution with distance of the high-frequency components of the jet noise spectrum. Experimental data provided by Howell and Morfey [1], although limited, strongly suggests that jet noise signals are sufficiently intense for nonlinear effects to be taken into account in their propagation to the distant field. More recently, Gee, Gabrielson, Atchley and Sparrow [2] have used F/A-18E static engine run-up measurements to argue that noise propagation is nonlinear in that case for military thrust conditions. It is clear that locally the nonlinear terms are, at all points, very small with respect to their linear counterparts, but, characteristically, the effect of the nonlinear terms is cumulative. Under fairly common conditions, even weak local nonlinearities might lead to significant distortion of the propagating waveform, and therefore of the spectrum as well. It was found by Howell and Morfey [3] that linear acoustic theory can only predict the propagation of noise fields accurately at low engine levels; at high power conditions, over distances ranging from 500 to 1000 m, the high-frequency attenuation (between 5 and 10 kHz) was observed to be 10 dB lower than expected. These distances are typically those used for aircraft noise certification measurements, and although the energy in this part of the spectrum contributes little to the overall sound pressure level (OASPL), the range of frequencies

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involved plays a particularly important part in determining perceived noise levels (PNL). In this paper, however, we do not incorporate the effects of ground impedance or of interference between outgoing paths and ground-reflected paths, which would need to be taken into account for noise certification. We assume, then, that these effects can be considered separately to those elaborated upon in this paper, and we refer the reader to Attenborough [4] for a discussion on the subject of ground impedance, and to Rendón and Coulouvrat [5] for an analysis of the interference between direct and reflected paths associated to a high-intensity source.

There are a variety of linear mechanisms which could be at least partly responsible for this under-prediction, such as temperature and humidity variations in the atmosphere, or any effects associated with thrust-dependent directivity patterns. We should also consider any inaccuracies in the measurement of the high-frequency spectrum, an especially difficult business, but Howell [6] has shown that we would still lack sufficient elements to account for this discrepancy. Our assumption, following Pernet and Payne [7], is that the anomalously high levels of the high-frequency components of the spectrum are to be accounted for by an energy transfer from the intense low-frequency components achieved through cumulative waveform distortion. The subsequent errors in prediction and control of the high frequencies associated with the use of linear acoustic theory have several implications for the jet noise problem, besides the invalidation of all linear scaling laws. First, measurements of flyover noise corrected to standard atmospheric conditions, but without consideration of absolute noise levels may be tainted by nonlinear propagation effects. Also, several methods are used to control high-frequency emissions from aircraft engines, such as engine tailpipe liners, but if these high frequencies are the result of nonlinear effects accumulating over long distances, any reduction of these frequencies at the source might prove to be less useful than previously thought.

Many experiments with finite-amplitude noise have been developed in an attempt to predict the nonlinear propagation of this noise. Pernet and Payne [7] measured high-intensity acoustic propagation in tubes, both for sinusoidal and noise signals, and produced a model which, however, does not allow for dispersion and is not really suited to the study of broadband signals. Bjørno and Gurbatov [8] also perform an experimental study of high-intensity acoustic noise in air-filled tubes, but their main interest is to investigate the power-law decay of the spectrum. In their experiment, which is of a more predictive nature, Webster and Blackstock [9] use an initial waveform which has been reconstructed from the initial power spectrum, with random phases assigned to each frequency component. This waveform is then propagated numerically using a scheme developed by Pestorius and Blackstock [10], and the power spectrum is then calculated as a function of distance.

Gurbatov et al. [11] has singled out two processes which in some way characterize nonlinear propagation of broadband noise: waveform steepening, and an increase in the time-scale of oscillations. The steepening leads to shock formation, and consequently to the energy transfer to high frequencies which we have previously discussed. The second process has to do with relative velocities of different shock fronts, and in particular with their coalescence. In this way, energy is transferred to the lower end of the spectrum. The spectrum broadens as a result of the combination of these two processes. The evolution of high-intensity broadband noise has been well studied for two limiting cases: at small distances, where the influence of the (at that moment) few shock fronts can largely be ignored, and at very large distances, where the shocks determine the statistical properties.

For small distances, well before shock formation, a Taylor series expansion for the power spectrum seems a reasonable way to implement a predictive scheme. This is the approach followed both by Crighton and Bashforth [12] and Howell and Morfey [1], in expansions which in both cases are well suited to describe the evolution of broadband noise signals, and which allow for different forms of atmospheric attenuation. They also share some disadvantages, such as the effect truncation of the series has on the accuracy of the expansions as range increases. The previous results also suggest that the near-linear approximations used do not properly describe the high frequencies, as this is an area of the frequency spectrum where large changes due to nonlinearity are to be expected. In these circumstances, Scott [13] has provided a correction for the high frequencies at small distances, as well as a more general asymptotic form for the high-frequency spectrum, assuming (in the same way that Bjørno and Gurbatov do [8]) that wherever shocks exist they will largely determine the high-frequency behaviour, provided the initial signal is smooth enough. Nonetheless, these results are of limited practical applicability with respect to the aircraft noise problem since in both cases the Reynolds number is taken to be either very large (Bjørno and Gurbatov) or infinite (Scott). In the following sections, we will take the Crighton–Bashforth model as our starting point, and with some modification use it as a more accurate predictive tool for small distances up to the moment when shocks determine the evolution of the noise spectrum.

2. A weakly nonlinear perturbation scheme

Since linear theory is almost universally used to describe the propagation of noise signals, we assume nonlinear propagation effects can be studied through a weakly nonlinear perturbation approach; nonlinear effects are dealt with as perturbations which in some sense are small. This type of approximation will have the advantages of simplicity and flexibility, and will prove useful in obtaining specific predictions to help clarify the role of nonlinear effects; in the scheme that we present here we will also strive for maximum transparency, and we will deal with broadband signals rather than harmonic waves.

Our starting point is the nonviscous Burgers equation, a model equation for plane, unidirectional wave propagation in a lossless gas, given as