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Fourier analysis of complex impedance (amplitude and phase) in nonlinear systems: A case study of diodes

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ABSTRACT

The general framework of Fourier analysis of complex impedance including both the amplitude and phase information is presented. For the single frequency sine/cosine input, Fast Fourier Transform can be used to obtain the amplitude and phase of the harmonics in the output signal. Complex nonlinear least squares fitting can then be applied to correlate the experimental and calculated results as in conventional impedance spectroscopy. General purpose *p*–*n* junction diodes were used as model systems to test the method as well as to compare it to other means of analysis.

1. Introduction

Broadly speaking, impedance measurement refers to the technique of applying electrical (current or voltage) sine/cosine (denoted as sine thereafter) wave(s) to the system and measuring the corresponding voltage or current response. The input signal can be a single frequency sine wave or multiple sine waves with different frequencies. The output signal can also be single or multiple sine waves, depending on the nature of input and whether the system is linear or not. So there are four possible types of combinations: (i) single sine input and single sine output, (ii) multiple sine input and single sine output, (iii) single sine input and multiple sine output and (iv) multiple sine input and output. A schematic of the four cases is shown in Fig. 1. The left two boxes represent the single and multiple frequency input waveform and the right two boxes represent the single and multiple frequency output waveform.

Fourier analysis refers to the analysis of functions or signals using Fourier series of trigonometric functions. Sometimes it is also called harmonic analysis since the trigonometric functions contains components that are integer multiple of the fundamental one. It is also called frequency response analysis because trigonometric waves of different frequencies are analyzed. The use of Fourier analysis comes naturally in impedance measurement since the input signal is made of single or multiple trigonometric waves as mentioned above.

The classical impedance measurement corresponds to case (i) for a small amplitude input. This is the conventional definition of impedance/dielectric spectroscopy if the frequency of sine input is swept in a range [1,2]. When it is applied to electrochemical systems, it is usually called electrochemical impedance spectroscopy. In cases (ii) and (iv), sine waves of multiple frequencies are applied at the same time. If the frequency distribution of these multiple sine waves is random, this is usually called the white noise method and is mainly for the purpose of saving measurement time [1,2]. Sometimes only two different frequencies are applied and this is called frequency modulation/intermodulation [3,4]. Actually most of the analog sine waveform generators cannot generate pure single frequency sine wave and it intrinsically contains harmonic (integer multiple of the intended frequency) sine waves [5]. The reason behind it is that these analog sine generators have their signal origins in the form of triangular waves, which are made of odd harmonics [5,6]. However, with the advancement of Direct Digital Synthesis (DDS) [7], low-noise monochromatic sine wave can be generated at a nonprohibitive cost.

Whether the input is of single or multiple frequency nature, if the system is nonlinear and the input amplitude is large, response signal will always contain information at different frequencies. This is generally called nonlinear impedance measurement and it becomes more popularly used in fuel cells [8,9], batteries [10], dielectrics [11], corrosion [12]. However, most off-the-shelf impedance analyzers can only provide information of the signal at the same frequency as the input, due to the use of the analog circuit or limited computation capability of earlier Digi-

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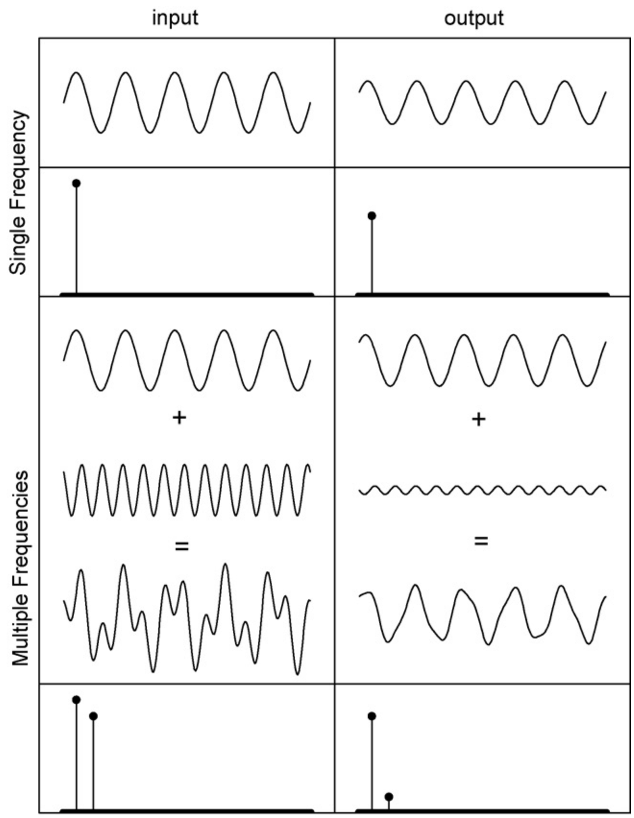

Fig. 1. Schematic of input and output signals encountered in the impedance measurement. The left two boxes show the single and multiple frequency input and the right two boxes show the single and multiple frequency output. The amplitude spectrum obtained from FFT is shown below the corresponding waveform.

tal Signal Processors (DSP). More and more turn-key instruments are now equipped with higher performance on-board Fast Fourier Transform (FFT) function to analyze all the frequency information and this makes the study of cases (iii) and (iv) easily accessible. The amplitude spectrum obtained from FFT for the corresponding waveform is shown in Fig. 1 for four different boxes, respectively.

It is to be noted that FFT based Fourier analysis has been mainly discussed under the context of multiple frequency input and single frequency output, i.e. case (ii), in the literature [1,2]. There is only a limited amount of work regarding the response analysis of nonlinear systems subjected to a single frequency perturbation [10,12–17]. In some of these works, the focus was still on the output of the same frequency as that of the input. So it is mainly an extension of case (i) at larger amplitudes. In addition, the discussion in these works was almost exclusively focused on the amplitudes of the signals with almost no discussion on the phase information, just as in Fig. 1 with only the amplitude spectrum being shown. In this work, the numerical Fourier analysis of both the amplitude and phase information concerning case (iii), i.e., single sine input and multiple sine output will be performed. The motivation is to establish a general framework to treat impedance data out of practical nonlinear systems, using diodes as model systems to demonstrate the feasibility of the approach. The diodes have the clear advantage of being robust and stable and will serve as the foundation for the investigation of other nonlinear behaviors such as interfaces in electrochemical systems, e.g., the charge transfer reaction described by the empirical Butler–Volmer equation [18].

2. Theory

2.1. Fourier analysis of general systems

The general investigation of a system is to find the correlation between the input signal $x(t)$ and output signal $y(t)$, where the input can be electrical, mechanical or optical, etc. If the system can sustain a steady state (time-invariant), a steady-state input and output x_{SS} and y_{SS} will be recorded. In impedance measurements, a perturbation of single or multiple sine waves are superimposed onto the steady-state input as

$$x(t) = x_{SS} + \sum_{n=1}^{M} c_n \cos(\omega_n t + \varphi_n) \tag{1}$$

where ω_n, c_n and ϕ_n are the angular frequency, amplitude and phase angle of the cosine wave. M is the number of cosine waves. If it is a single frequency perturbation, M is equal to 1. Here the cosine wave instead of sine wave is used for simplicity. The goal of impedance measurement is to investigate the system properties by studying the correlation between the input as in Eq. (1) and the corresponding output. As mentioned in the introduction, a single frequency input is the focus of this work so we have

$$x(t) = x_{SS} + \Delta x_{SS} \cos(\omega t + \varphi) = x_{SS} + \Delta x_{SS} \cos\theta(t) \tag{2}$$

where Δx_{SS} is the amplitude and $\theta(t)$ is used for convenience.

In practical experiments, both the input and output signals are sampled as discrete data points in a finite time window. For this cosine wave with angular frequency of ω, usually integer numbers of its period is used as the collection time. For simplicity, only one period of data is collected so the measurement time is from 0 to $2\pi/\omega$. The output signal can be expanded as Fourier series in this time interval and Fourier coefficients are the Fourier transform of the signal. Three equivalent forms can be used. The first form of Fourier series can be written as

$$y(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(n\omega t) + b_n \sin(n\omega t)] \tag{3}$$

The Fourier coefficients have the form of

$$a_0 = \frac{\omega}{\pi} \int_0^{2\pi/\omega} y(t) dt \tag{4}$$

$$a_n = \frac{\omega}{\pi} \int_0^{2\pi/\omega} y(t) \cos(n\omega t) dt \tag{5}$$

$$b_n = \frac{\omega}{\pi} \int_0^{2\pi/\omega} y(t) \sin(n\omega t) dt \tag{6}$$

The second form of Fourier series can be written as

$$y(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} A_n \cos(n\omega t + \phi_n) \tag{7}$$

which is related to the first form by

$$a_n = A_n \cos\phi_n \tag{8}$$

$$b_n = -A_n \sin\phi_n \tag{9}$$

The third equivalent form of Fourier series can be written as

$$y(t) = \sum_{n=-\infty}^{\infty} B_n e^{jn\omega t} \tag{10}$$