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Optimal control application to an Ebola model

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ABSTRACT

Ebola virus is a severe, frequently fatal illness, with a case fatality rate up to 90%. The outbreak of the disease has been acknowledged by World Health Organization as Public Health Emergency of International Concern. The threat of Ebola in West Africa is still a major setback to the socioeconomic development. Optimal control theory is applied to a system of ordinary differential equations which is modeling Ebola infection through three different routes including contact between humans and a dead body. In an attempt to reduce infection in susceptible population, a preventive control is put in the form of education and campaign and two treatment controls are applied to infected and late-stage infected (super) human population. The Pontryagins maximum principle is employed to characterize optimality control, which is then solved numerically. It is observed that time optimal control is existed in the model. The activation of each control showed a positive reduction of infection. The overall effect of activation of all the controls simultaneously reduced the effort required for the reduction of the infection quickly. The obtained results present a good framework for planning and designing cost-effective strategies for good interventions in dealing with Ebola disease. It is established that in order to reduce Ebola threat all the three controls must be taken into consideration concurrently.

1. Introduction

The principal aim of modeling infectious diseases is to be able to make judicious decisions in the application of control interventions of the infection to eliminate and ideally to eradicate it from the human population. Simulations and modeling can optimize control efforts such that limited resources are targeted to achieve the highest impact [1]. The aim of this paper is to review the epidemiology of the Ebola pandemic and discuss the optimal control model that governs the spread of the virus in human population and suggest the optimum control strategies to control and curb the spread in the future. The world witnessed an unprecedented Ebola outbreak in West Africa which in the end was reported in some parts of Europe and North America [2]. By December 13th 2015 there had been confirmed reported cases in excess of 28 600 in total with over 11 000 people losing their lives particularly in West Africa and to a lesser extent elsewhere in the world. Outside Africa; Italy, Spain, the United Kingdom and United States of America [2] were also affected albeit with no case fatalities except for one in the USA (Table 1).

The outbreak has been acknowledged by the World Health Organization as a Public Health Emergency of International Concern. The three West African countries (Guinea, Sierra Leone and Liberia) by far been hardest hit now account for about 99.9% of all infections and deaths. These countries are known to have only recently emerged from long periods of conflict and instability and thus have weak health systems, human and infrastructural resources [3].

Ebola is known to be transmitted to humans via contact with bodily fluids and secretion of infected animals mainly fruit bats,

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Table 1
Ebola situation report 13th December 2015 [2].

Country	Cases	Death	% Mortality
Guinea	3807	2536	67
Liberia	10666	4809	45
Sierra Leone	14122	3955	28
Italy	1	0	0
Mali	8	6	75
Nigeria	20	8	40
Senegal	1	0	0
Spain	1	0	0
UK	1	0	0
USA	4	1	25
Total	28640	11315	40

monkeys, porcupines, forest antelope and the like. It is thereafter spread from person to person through direct contact with infected persons [2,4,5]. The incubation period beyond which infected people become symptomatic has been estimated to range between 2 and 21 days [6]. The Ebola virus is a unique virus having a filamentous, enveloped non-segmented negative-sense RNA. It belongs to the genus *Ebolavirus*, within the family of Filoviridae. Its envelope glycoprotein facilitates the entry of the virus to living cells [7,8]. Till date five strains of the virus are known: Zaire, Sudan, Tai Forest, Bundibugyo, and Reston with Zaire stain being the most virulent with up to 90% fatalities [9]. The Pasteur Institute (Lyon, France) has sequenced the viral strain currently under circulation in West Africa and reports a strong homology of 98% with *Zaire ebolavirus* (the most virulent). The origin of the Ebola virus has been somewhat unclear [10,11]. However, in 2005, Leroy and co-workers reported in the Nature, evidence of asymptomatic infection by Ebola virus in three species of fruit bat [12]. This indicated that fruit bats belonging to the family Pteropodidae are the natural host of the Ebola virus. Ebola viral disease has no effective treatment or vaccines, currently only supportive care can be given to patients.

Emerging tropical infectious diseases have been persistent in causing untold economic hardships to relatively poor countries with weak health systems. The overarching goal of public health is to reduce disease burden by curtailing transmission or mitigating its severity. There are at least two fundamental public health guiding principles that exist to manage the spread of an infectious disease like Ebola viral disease that has no effective vaccines or treatment. These are (i) effective isolation of persons with symptoms and (ii) tracing the contacts of symptomatic cases for clusters of exposed persons and quarantining them for monitoring [13].

Mathematical models have played a vital role in the dynamics and control of many epidemics including malaria, severe acute respiratory syndromes and Ebola [14].

Some previous models of Ebola virus, especially the predictive models endeavour to calculate a threshold called basic reproductive number R0. The dynamics of transmission of the disease has been analyzed in terms of the reduction of the basic reproductive number [15–18].

However, all these models fail to take into account time dependent control strategies and all their discussions have been concentrated on prevalence of the disease at equilibra. Time dependent control has been employed in the study of dynamics of diseases. For example, Rachah and Torres, 2015 investigated the effect of vaccination on a proportion of susceptible population and observed that the rate of infection of Ebola reduced due to this intervention [19]. Chowell and Nishiura 2014 also applied time optimal control to study Ebola epidemic reduction [14]. Other studies on using time optimal control to provide appropriate interventions to minimize the spread of diseases have also been carried out successfully [20–23]. This technique of studying control strategies present an enviable theoretical results that can assist in providing tools for designing epidemic control programmes.

In this work, time dependent optimal control is explored and considered which deals with both “case holding” and “case finding” on Ebola model proposed by Rivers *et al.* [24]. The model assumes that there is a difference between first-stage infections and late-stage infection called super infection. Their model further assumes individuals in the latent stage develop active Ebola infection at a given rate. It also assumes that a proportion of both first and late stage-infection (super infection) recover and others move to death compartment. We present three control mechanisms which comprise two “case finding” and a “case holding” in the model. The “case finding” is usually made up of activities that lead to preventive measures including screening, public education and others. The “case holding” also has to do with designed activities that ensure patients take their drugs within stipulated times so that they are cured. The first case finding is incorporated by adding a control term that characterizes the contact between susceptible and infectious individuals so that the rate of infection will be reduced. The second case finding is instituted in the model by adding a control term that identifies proportions of those individuals in the latent stage or exposed to the disease and cure them so that the rate of getting the disease will be reduced. The holding is incorporated in the model by adding a control term that may minimize the treatment failure rate of individuals with Ebola disease. We choose the reduction of infected individuals of Ebola to be our main objective having a lower cost of the controls.

The paper is arranged as follows: section 2 is devoted to describing an Ebola model with three control terms incorporated. In addition, the objective function is introduced in this section. In section 3, the analysis of optimal control is discussed. Section 4 contains some numerical studies of optimal controls. Finally, section 5 deals with the conclusions of the studies.

2. Ebola disease model

The model structure is presented in Figure 1 and the state system of the Ebola model is the following six nonlinear ordinary differential equations proposed by Pontryagin *et al.* [25] and slightly modified:

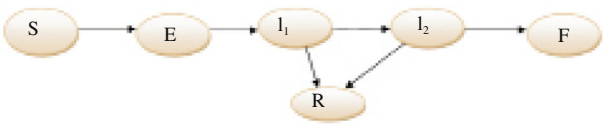

Figure 1. Stage-structured compartmental model of Ebola virus disease, which splits the population into susceptible (S), exposed (E), first-stage infected (I_1), late-stage infected (I_2), recovered (R), and funeral transmissible (F).
Red compartments are transmissible, and recovery rates are greater from I_1 than from I_2 [25].