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Nonlinear stability of the motionless state of thermosolutal Rivlin–Ericksen fluid in porous medium

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Abstract

Energy method is used to study the nonlinear stability of the motionless state of thermosolutal Rivlin–Ericksen fluid in porous medium for stress-free boundaries. By defining energy functionals we will show that for $\tau = (E'P_C)/(EP_T) \le 1, \hat{\alpha} = (C/R) \ge 1$ the motionless state is always stable and for $\tau \le 1, \hat{\alpha} < 1$ the sufficient and necessary conditions for stability coincide, where P_C, P_T, C and R are the Schmidt number, Prandtl number, Rayleigh number for solute and heat, respectively, E' and E are two constants related to porosity of porous medium. Unlike the energy-decay rate in previous works concerning the nonlinear stability of Bénard problem for the same boundaries, this quantity in present work is completely independent of mode numbers.
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1. Introduction

In recent years, the investigation of thermosolutal convection in fluids in a porous medium has attracted attention of many researchers because of its applications in geophysics, astrophysics, soil science, ground water hydrology and recovery of crude oil from the pores of reservoir rocks. In the case of non-Newtonian fluids the study of the thermosolutal convection problems has also gained more and more importance owning its applications in chemical technology, petroleum industry and composite materials. For instance, there are polymeric materials that are used in the manufacture of semi-conductor devices that need the solidification processes. To make these materials free from defect, convection must be controlled. However, the investigations of the nonlinear stability of flows through porous media in non-Newtonian fluids are very few in numbers as compared to those in Newtonian fluids. This is just the motivation of present work.

According to Sharma et al. [1] on neglecting the squares and products of A_2 in the constitutive equations characterizing Rivlin–Ericksen fluid we have

$$\tau_{ij} = -p\delta_{ij} + \nu A_1 + \nu' A_2 + \nu'' A_1^2$$

where τ_{ij} is the stress tensor, p the pressure, δ_{ij} the Kronecker delta, and ν, ν' and ν'' are three measurable material constants. They denote respectively the viscosity, elasticity and cross-viscosity, A_1 and A_2 are given by

$$A_{ij}^{(1)} = (u_{i,j} + u_{j,i})$$

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and

$$A_{ij}^{(2)} = \frac{\partial}{\partial t}A_{ij}^{(1)} + u_m A_{ij,m}^{(1)} + A_{im}^{(1)}u_{m,j} + A_{mj}^{(1)}u_{m,i}$$

For fluid permeating a porous material, on the basis of Darcy's law the usual viscous term in the equations of Rivlin–Ericksen fluid motion is replaced by the resistance term $[-1/k_1(\nu + \nu'(\partial/\partial t))\boldsymbol{u}]$, where k_1 is the medium permeability and $\boldsymbol{u}$ is the Darcian velocity of the fluid.

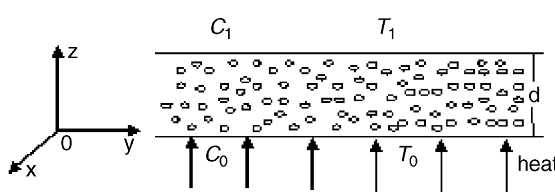
An infinite, horizontal, incompressible Rivlin–Ericksen fluid layer, in porous medium, heated and soluted from below for stress-free boundary conditions is considered in this paper. The linear stability of motionless state of this system with solute concentration, uniform rotation and uniform vertical magnetic field is studied in Sharma et al. [1], the linear stabilizing effect of solute concentration is proved. As we know, the method of linearized stability tends to overestimate stability, the stability so obtained is only for indefinitely small perturbations and global stability cannot be derived. Therefore, different methods like the energy method have been devised, which enable us to obtain unconditional stability or, at least, conditional stability with finite amplitude. In this paper we consider only the effect of solute concentration. By applying generalized energy method (see Xu [2], Galdi and Padula [3], Xu and von Wahl [4] and Mulone and Rionero [5]) and through defining energy functional we shall prove the sufficient and necessary condition for stability coincide, for the case $\hat{\alpha} < 1$ and $\tau \leq 1$. This result was also proved in previous paper by Xu [2]. Moreover, in this paper we shall take advantage of a mathematical decomposition and transfer the system to an equivalent one, then we introduce two balance fields instead of only one in Xu [2], so that, for the case $\hat{\alpha} \geq 1$ and $\tau \leq 1$, we can directly show that the nonlinear stability boundary $R_{\mathrm{E}} = +\infty$, that means the basic state is always stable in this case.

Unlike previous works concerning the nonlinear stability of the Bénard problem with rotation (solute concentration or magnetic field) (see Galdi and Padula [3], Mulone and Rionero [5] and Kaiser and Xu [6]) for free boundaries, where the decay rate and stability ball depend on the mode numbers in the horizontal directions and these quantities shrink to zero for vanishing mode numbers, in the present paper, these quantities for our problem are completely independent of the mode numbers.

2. Mathematical formulation and perturbation equations

Let us consider an infinite horizontal incompressible Rivlin–Ericksen layer $\mathbb{R}^2 \times (0, d)$ in a Cartesian reference frame Oxyz with unit vector $\boldsymbol{k} = (0, 0, 1)$ for the z-axis, which is parallel to the direction of the vertical. The layer is heated and soluted from below so that the temperature and solute concentration at $z = 0$ are T_0 and C_0, and at $z = d$ are T_1 and C_1, respectively. The system is subject to the force of gravity $\boldsymbol{g} = -g\boldsymbol{k}$. Moreover, we assume that the fluid is flowing through an isotropic and homogeneous porous medium of porosity ϕ and medium permeability k_1. In this configuration the motionless state is described by

$$\boldsymbol{u} = 0, T = -\beta_T z + T_0, C = -\beta_C z + C_0$$



Let $\boldsymbol{u} = (u, v, w), \theta, \gamma, p$ represent the perturbations of the velocity, temperature, solute concentration and pressure fields to the motionless state, respectively. Then after non-dimensionalizing the equations of Rivlin–Ericksen fluid (see Sharma et al. [1]), we obtain the perturbation equations with $z \in (-(1/2), 1/2)$:

$$\begin{cases} \phi^{-1}\partial_t \boldsymbol{u} = -P_l^{-1}(1 + F\partial_t)\boldsymbol{u} + (R\theta - C\gamma)\boldsymbol{k} - \nabla p - \phi^{-2}(\boldsymbol{u} \cdot \nabla \boldsymbol{u}) \\ \nabla \cdot \boldsymbol{u} = 0 \\ EP_T\partial_t\theta = -(-\Delta)\theta + Rw - EP_T\boldsymbol{u} \cdot \nabla\theta \\ E'P_C\partial_t\gamma = -(-\Delta)\gamma + Cw - E'P_C\boldsymbol{u} \cdot \nabla\gamma \end{cases} \tag{1}$$

where R^2(Rayleigh number for heat)$=(\alpha_T\beta_T g d^4)/(\kappa\nu)$, C^2(Rayleigh number for solute)$=(\alpha_C\beta_C g d^4)/(\kappa'\nu)$, P_T(Prandtl number) $= \nu/\kappa, P_C$(Schmidt number) $= \nu/\kappa'$, $P_l = k_1/d^2$, $F = \nu'/d^2$. Here $\alpha_T, \alpha_C, \beta_T, \beta_C, \nu, \nu', \kappa, \kappa'$ and k_1 denote, respectively, the thermal coefficient of expansion, the solvent coefficient of expansion, the temperature gradient, the solute gradient, the kinematic viscosity, the kinematic viscoelasticity, the thermal diffusivity coefficient, solute diffusivity coefficient and medium permeability. E and E' are two constants corresponding to heat and solute, which also depend on porosity ϕ. Δ is the Laplace operator.

$\boldsymbol{u}, \theta, \gamma$ and p are assumed to be x, y-periodic with respect to a rectangle $\mathcal{P} = (-(\pi/\alpha), (\pi/\alpha))(-(\pi/\beta), (\pi/\beta))$ with wave numbers α, β in x- and y-direction, respectively.

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