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Numerical study of the shape parameter dependence of the local radial point interpolation method in linear elasticity

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HIGHLIGHTS

- The LRPIM is derived from the local weak form of the equilibrium equations for solving a thin elastic plate.
- The method LRPIM is used the trial and test functions in the weak form.
- Convergence of the LRPIM depends on number of parameters derived from local weak form and sub-domains.
- The effect of distributions nodes number by varying nature of material and the RBF-TPS.
- The calculated results are compared with the analytical solution of the deflection.

ABSTRACT

The method LRPIM is a Meshless method with properties of simple implementation of the essential boundary conditions and less costly than the moving least squares (MLS) methods. This method is proposed to overcome the singularity associated to polynomial basis by using radial basis functions. In this paper, we will present a study of a 2D problem of an elastic homogenous rectangular plate by using the method LRPIM. Our numerical investigations will concern the influence of different shape parameters on the domain of convergence,accuracy and using the radial basis function of the thin plate spline. It also will presents a comparison between numerical results for different materials and the convergence domain by precising maximum and minimum values as a function of distribution nodes number. The analytical solution of the deflection confirms the numerical results.
The essential points in the method are:

- The LRPIM is derived from the local weak form of the equilibrium equations for solving a thin elastic plate.
- The convergence of the LRPIM method depends on number of parameters derived from local weak form and sub-domains.
- The effect of distributions nodes number by varying nature of material and the radial basis function (TPS).

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Method detail

Meshless method has attracted more and more attention from researchers in recent years.It is regarded as a potential numerical method in computational mechanics. Several meshless methods, such as smooth particle hydrodynamics (SPH) method [1,2], element free Galerkin (EFG) method [3], meshless local Petrov-Galerkin (MLPG) method [4–8], point interpolation method (PIM) [9,10], local point interpolation method (LPIM) [11] and local radial point interpolation method (LRPIM) has been proposed by Liu et al. [10,12,13]. In LRPIM, the point interpolation using the radial basis function to construct the shape functions which have the delta function property. The radial basis function (RBFs) is thin plate spline (TPS) [14,15]. Local weak forms are developed using weighted residual method locally from the partial differential equation of linear elasticity of 2D solids. The number of numerical examples will be presented to demonstrate the convergence and accuracy, validity and efficiency of the present methods. The local radial point interpolation method LRPIM is a Meshless method with properties of simple implementation of the essential boundary conditions and with the lower cost than moving least squares (MLS) methods.

This paper deals with the effect of sizing parameter of subdomains on the convergence and accuracy of the methods. Numerical values will be presented to specify the convergence domain by precising maximum and minimum values as a function of distribution nodes number and by using the radial basis functions TPS (thin plate spline). It also will present a comparison with numerical results for different materials. The analytical solution of the deflection confirms the numerical results. The LRPIM method will be developed to solve the problem of a thin elastic homogenous plate. The local weak form and numerical implementation are presented in section 3, numerical example for 2D problem are given in section 4. Then, the paper will end with the results, discussions and finally the conclusions.

RPIM shape functions in meshless method

$u^h(x)$ is composed of two parts: $P_j(x)$Polynomial basis functions and $R_i(x)$ the radial basis functions RBFs [16–20]:

$$u^h(x) = \sum_{i=1}^{n} R_i(x)a_i + \sum_{j=1}^{m} P_j(x)b_j \tag{1}$$

n is the number of field nodes in the local support domain and **m** is the number of polynomial terms.

Radial basis is a function of distance **r:**

$$r = \sqrt{(x - x_i)^2 + (y - y_i)^2} \tag{2}$$

The above Eq. (1) can be expressed in the matrix form [17]

$$\mathbf{U_1} = \mathbf{R} \times \mathbf{a} + \mathbf{P} \times \mathbf{b} \tag{3}$$

where $\mathbf{U_1}$ is the vector of function values: $\mathbf{U_1} = [u_1,\ u_2,\ u_3, \ldots,\ u_n]^T$

R The moment matrix of RBFs, **P** is the moment matrix of Polynomial basis function and **a, b** are the values of unknowns coefficients (Radial and Polynomial).

We note that, to obtain the unique solutions of Eq. (2), the constraint conditions should be applied as follows [21]:

$$\sum_{i=1}^{n} P_{ij}(x)a_i = 0 \quad j = 1, 2, \ldots, m \tag{4}$$

the combining of Eqs. (3) and (4) yields a set of equations in the matrix form:

$$\overline{\mathbf{U}}_1 = \begin{bmatrix} \mathbf{U}_1 \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & \mathbf{P} \\ \mathbf{P^T} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} = \mathbf{Ga_0} \tag{5}$$