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ORIGINAL RESEARCH ARTICLE

On the nonlinear internal waves propagating in an inhomogeneous shallow sea

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Summary A concept of conservation of energy flux for the internal waves propagating in an inhomogeneous shallow water is examined. The emphasis is put on an application of solution of the Korteweg—de Vries (KdV) equation in a prescribed form of the cnoidal and solitary waves. Numerical simulations were applied for the southern Baltic Sea, along a transect from the Bornholm Basin, through the Słupsk Sill and Słupsk Furrow to the Gdańsk Basin. Three-layer density structure typical for the Baltic Sea has been considered. An increase of wave height and decrease of phase speed with shallowing water depth was clearly demonstrated. The internal wave dynamics on both sides of the Słupsk Sill was found to be different due to different vertical density stratification in these areas. The bottom friction has only negligible influence on dynamics of internal waves, while shearing instability may be important only for very high waves. Area of possible instability, expressed in terms of the Richardson number *Ri*, is very small, and located within the non-uniform density layer, close to the interface with upper uniform layer. Kinematic breaking criteria have been examined and critical internal wave heights have been determined.

1. Introduction

A sea bottom is only seldom horizontal and usually water depth and vertical density structure are varying in space and time. Observations of internal waves in the Andaman Sea, Sulu Sea, Australian North West Shelf and the South China Sea, as well as in other sea basins, show that shoaling effects and local bottom changes may influence essentially the

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internal wave evolution. At present, the South China Sea is known as a "hot spot" for observations of the internal waves generated by tides in deep sea and propagating on the ocean shelf (Lien et al., 2014). A large-amplitude depression of the first mode of the internal solitary waves has been observed during spring tide using both shipboard and mooring ADCP and CTD measurements. Maximum negative vertical displacement approached the value of 100—150 m, nearly half of the water depth, and observed phase velocity was equal to about 2 m s^{-1}. Comprehensive observations of the internal tides by Holloway (1994, 1996) in a region of shelf-break on the Australian North West Shelf showed that internal tides exhibit a three-dimensional structure. Waves of depression are observed during summer, while in winter they are weaker and are waves of positive elevation.

Satellite SAR images provided an excellent tool for observation and recording of the internal waves in the ocean. The Indonesian Throughflow, the Middle Atlantic Bight, the Gulf of Aden and the White Sea are only a few examples of such locations. Other examples have been collected and discussed by Massel (2015).

Except the observations in nature, several attempts of modelling of the internal waves have been reported. Vlasenko and Hutter (2002) using numerical simulations, studied transformation of large amplitude internal solitary waves over a slope-shelf topography. Grimshaw et al. (2004) employed the extended Korteweg-de Vries (eKdV) equation to simulate propagation of internal solitary waves taking into account a real variability of wave parameters for several oceanic shelves. It was shown that if the background environment varies sufficiently slowly in comparison with an individual solitary wave, then the wave has a soliton-like form with varying amplitude and phase for large distances.

Combined effect of the Earth rotation and varying bathymetry on the solitary internal waves propagating on long distances was described by Grimshaw et al. (2014) using an extension of the KdV equation in the form of the Ostrovsky equation. The main finding of this study is that the Earth rotation induces a formation of a secondary wave packet, trailing behind the leading wave. These results correspond to bottom topography and density stratification for the cross section on the South China Sea. However, as the authors argued, they are rather typical for many other continental slopes.

In contrast to the deep sea there are not numerous papers on the internal waves dynamics in the shallow water. The application of the concept of conservation of energy flux to study the long internal wave dynamics in the horizontally inhomogeneous ocean was reported by Pelinovsky and Shavratsky (1976) and Pelinovsky et al. (1994), however without any connections to the real bathymetry and density stratification. Laboratory experiments and theoretical studies have been conducted by Helfrich and Melville (1986) and Helfrich (1992) to explore shoaling of the internal solitary waves of depression in a two-layer system on a uniform slope. An extended Korteweg-de Vries (eKdV) equation, including the nonlinearity, dispersion and dissipation was solved numerically for single and rank-ordered pairs of solitary waves incident on the slope-shelf topography of large dimension when the topographic effects dominate nonlinearity and dispersion. The authors discussed an application of the developed theoretical models for the real oceanographic situations, however expressing some doubts to which extent a laminar damping in the laboratory tanks properly reflects turbulent eddy viscosity in the real ocean.

In this paper, numerical simulations of long internal waves motion over a slowly changing bathymetry and density stratification in the southern Baltic Sea are considered. For analysis, the typical temperature and salinity vertical structure, recorded during the cruise of the research vessel s/v *Oceania* in February 2003 along the transect from Bornholm Basin, through Słupsk Sill and Słupsk Furrow to Gdańsk Basins (see Fig. 1) was used. This period corresponded to one of the major inflows of saline water from the North Sea to the Baltic Sea and high dynamics of the pycnocline motions (Massel, 2015; Piechura and Beszczyńska-Möller, 2004).

A concept of the energy flux conservation was considered under the assumption that the internal wave maintains its cnoidal-like shape with varying wave parameters. Also, the limiting cases of the cnoidal wave, namely the solitary and sinusoidal waves are taken into account. In the numerical simulations, the non-dissipative motion, as well as motion with several dissipative mechanisms, such as bottom friction, shearing instability with mixing and wave breaking, have been taken into account.

The paper is organised as follows. In Section 2, the concept of energy flux conservation for internal waves is introduced. In Section 3, motion of the internal waves of prescribed form is discussed and governing equations are solved. Finally, variation of the wave height and wave shape are determined and illustrated for given locations along the transect in the southern Baltic Sea. The major conclusions are formulated in Section 4.

2. Concept of the energy flux conservation

We would like to consider a long internal wave motion in two-dimensional vertical plane (x, z) with z-axis positive upward. Water depth is slowly varying in the x direction and the refraction effects are omitted. The background density $\overline{\rho}(x,z)$ is a known slowly varying function of x and z coordinates. Under the Boussinesq approximation, the rate of wave energy change can be presented as follows (Kundu et al., 2016; Massel, 2015):

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\overline{\rho}(x,z)(u^2+w^2)\right]+g\rho(z)w+\nabla\cdot(pu)=0, \tag{1}$$

where u and w are the velocity components in x and z direction, respectively, p is the water pressure and $\rho(z)$ is the perturbation of density component due to wave action. The first term in Eq. (1) represents a rate of change of the kinetic energy and the second term can be considered as the rate of change of potential energy. The last term is the net work done by the pressure forces and it can be interpreted as the divergence of the energy flux pu (Gill, 1982).

Therefore, the energy flux integrated over water depth and averaged over wave period can be written as follows:

$$\overline{F_E(x)}=\frac{1}{T}\int_0^T\int_{-h(x)}^0 p(x,z,t)u(x,z,t)\,dz\,dt, \tag{2}$$

in which $T = 2\pi/\omega$ is the internal wave period and ω is the wave frequency.

To determine the energy flux $\overline{F_E(x)}$ we consider the vertical displacements of isopycnals due to long internal