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Proof phenomenon as a function of the phenomenology of proving

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ABSTRACT

Kurt Gödel wrote (1964, p. 272), after he had read Husserl, that the notion of objectivity raises a question: "the question of the objective existence of the objects of mathematical intuition (which, incidentally, is an exact replica of the question of the objective existence of the outer world)". This "exact replica" brings to mind the close analogy Husserl saw between our intuition of essences in *Wesensschau* and of physical objects in perception. What is it like to experience a mathematical proving process? What is the ontological status of a mathematical *proof*? Can *computer assisted provers* output a *proof*? Taking a *naturalized world* account, I will assess the relationship between mathematics, the physical world and consciousness by introducing a significant conceptual distinction between *proving* and *proof*. I will propose that *proving* is a phenomenological conscious experience. This experience involves a combination of what Kurt Gödel called *intuition*, and what Husserl called *intentionality*. In contrast, *proof* is a function of that process — the mathematical phenomenon — that objectively self-presents a property in the world, and that results from a spatiotemporal unity being subject to the exact laws of nature. In this essay, I apply phenomenology to mathematical proving as a performance of consciousness, that is, a lived experience expressed and formalized in language, in which there is the possibility of formulating intersubjectively shareable meanings.

> If you will stay close to nature, to its simplicity, to the small things hardly noticeable, those things can unexpectedly become great and immeasurable.
>
> —Rainer Maria Rilke, Letters to a Young Poet.

1. *Naturalizing* mathematics

Husserl wanted to ensure that basic categories employed by natural science were not thought to be products of some merely such contingent features. In fact, he tried to define the limits of what science, or naturalism could inform us of (Gallagher, 2012). He considered that "naturalism is a phenomenon consequent upon the discovery of nature … considered as a unity of spatiotemporal being subject to exact laws of nature. With the gradual realization of this idea in constantly new natural sciences that guarantee strict knowledge regarding many matters, naturalism proceeds to expand more and more" (Husserl, 1965, p. 79). Husserl was not opposed to natural scientific explanation; rather, he considered that an extreme naturalism in formal logic, mathematics, and ideal essences might lead to their reduction to psychological processes of the knowing subject. In his perspective, regarding an extreme version of naturalism, if our brain processes evolve over time (which they do), then the laws of nature may be different in the future. In other words, both psychological processes and laws of nature are subject to biological evolution. Nevertheless, Husserl considered that the results of transcendental phenomenology should not be ignored by science, as "every analysis of theory of transcendental phenomenology—including … the theory of the transcendental constitution of an objective world—can be developed in the natural realm, by giving up the transcendental attitude" (1970, §57).

Certain authors such as De Preester (2002) and Lawlor (2009) consider that naturalizing phenomenology is a contradiction in terms, since phenomenology is, by definition, non-naturalistic. Nevertheless, Merleau-Ponty's work (1942; 1945) seems to contain direct suggestions for naturalizing phenomenology. As reported by Merleau-Ponty, with science, one is expressive in relation to nature (1945, p. 391). This fundamentally changes Husserl's transcendental conception and shifts the focus from the transcendental ego to the body.

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Cognitive Science has grounded this view of the body as a decisive instance in bringing about behavioural and mental capacities (De Preester, 2002, p. 654). The introduction of phenomenology in cognitive science has challenged its basic assumptions and has brought a view that is more consistent with the views of Husserl and Merleau-Ponty concerning intentionality, intersubjectivity action, and embodiment.[1] Contemporary *embodied* cognitive science—contrary to its previous orthodox view—is grounded in the ecological-enactive approach (Bermudez et al., 1995; Clark, 1997; Varela et al., 1991). This perspective claims that cognition is best characterized as belonging to embodied, situated agents, i.e. agents who are in the world (Gallagher and Varela, 2003). In this approach, researchers in artificial intelligence and robotics, phenomenologists and philosophers of mind work together to advance an understanding of the embodied, ecologically situated and enactive mind.

For many authors the difficult question is how *naturalized phenomenology* can be accomplished without losing the specificity of phenomenology. In this regard, it is of great importance to consider what one means by *naturalization*. Naturalization can be defined as to "integrate into an explanatory framework where every acceptable property is made continuous with the properties admitted by the natural science" (Roy et al., 1999, pp. 1–2). Naturalization means, "not being committed to a dualistic kind of ontology" (1999, p. 19). Roy et al. (1999) propose a recategorization of phenomena at a level of abstraction necessary to acknowledge the common properties between phenomenological data and objective data developed in the sciences. As reported by this interdisciplinary group of researchers at the Centre de Recerche en Epistémologie Appliquée (CREA), "It is our general contention … that phenomenological descriptions of any kind can only be naturalized, in the sense of being integrated into the general framework of natural sciences, if they can be mathematized." (1999, p. 42). Accordingly, this idea involves a mathematical interpretation, i.e. a transformation of concepts into algorithms similar to transformations of this kind found in the physical sciences, through a formal language that expresses phenomenological findings. This appeal to mathematics demands formalization and intersubjective meaning verifiable within a common language that is clearly understood by science, namely, mathematics.

Husserl considered mathematical formula as incapable of capturing phenomenological results, as "one cannot define in philosophy as in mathematics; any imitation of mathematical procedure in this respect is not only unfruitful but wrong, and has most injurious consequences" (Husserl, 1976, p. 9). According to Roy et al. (1999), this may have been accurate in mathematics in Husserl's time; however, the development of dynamic systems theory offered new possibilities in this regard (p. 43). In fact, the opposition Husserl introduces between mathematics and phenomenology is "the result of having mistaken certain contingent limitations of the mathematical and material sciences of his time for absolute ones. In our opinion, it is indeed arguable that scientific progress has made Husserl's position on this point largely obsolete and that this *factum rationis* puts into question the properly scientific foundations of his anti-naturalism" (pp. 42–43). In other words, most of Husserl's scientific reasons for opposing naturalism have been invalidated by the progress of science (p. 54). In fact, as illustrated bellow, the editors claim that a genuine mathematical description of experiential consciousness is possible in the construction of a mathematical proof. Therefore one of the major impediments to the naturalization of phenomenology has been removed (pp. 55–56). The essential property in mathematical formalism is its exactness regardless of neurobiological or phenomenological facts (pp. 51, 68). The moment we are in the possession of a mathematical reconstruction of phenomenological descriptions, the only remaining problem is to articulate those reconstructions with the tools of relevant lower-level natural sciences (pp. 48, 63).

This proposal inspired by Marbach's work (1993) who, following Husserl's own proposal[2] for formal notation, suggested a formal symbolic language for phenomenology which developed a formalized notation, and assessed the question of whether it is possible for mathematics to capture the lived experience described by phenomenology. Marbach (1993, 2010) proposed that formalizing language can improve the possibility of formulating intersubjective shareable meanings.

2. Proving as a mathematical description of experiential consciousness

Part of Husserl's work was to provide an adequate phenomenological description of consciousness not contained within any well-established materialistic or naturalistic framework. Moreover, Husserl believed that a proper understanding of the conscious appropriation of the world would provide not only an understanding about consciousness but also about the world. Consciousness is, in his perspective, a place where the world can reveal and articulate itself. Phenomenology is concerned with transcendental subjectivity and not with empirical consciousness. Merleau-Ponty called for a redefinition of transcendental philosophy (1942, p. 241) that does not make us choose between either an external scientific explanation, or an internal phenomenological reflection: one does not unravel the relation between consciousness and nature (Zahavi, 2004). A redefinition that is beyond both objectivism and subjectivism. As reported by Merleau Ponty, "the ultimate task of phenomenology as philosophy of consciousness is to understand its relationship to non-phenomenology. What resists phenomenology within us—natural being, the 'barbarous' source Schelling spoke of—cannot remain outside phenomenology and should have its place within it" (1964, p. 178). In fact, Merleau-Ponty goes a step further since he considers that phenomenology can be changed and modified through its dialogue with the empirical disciplines. The theory of mind and cognition must begin with categories of things in the everyday common sense world—what Husserl called the *life-world*, that world which lies between quarks and the cosmos. This *life-world* is a horizon of all our experiences. In fact, it is that background on which all things appear as themselves and are meaningful. This *life-world* cannot, however, be understood in a purely static manner; but rather a dynamic horizon in which we *live*, and which "lives with us" in the sense that nothing can appear in our *life-world* except as *lived*.

A phenomenology of consciousness cannot begin the ontology of the world-around-us by dealing with bosons and black holes, or neurons and the neural nets, abstracting so far from our familiar concerns that we no longer know where we fit in.

From a phenomenological perspective, mathematics is a performance of consciousness, a mathematical experiential consciousness that involves the notion of *intentionality*. Brentano considered that every mental phenomenon contains the "intentional inexistence" of an object toward which the mental phenomenon is directed. From his perspective, identifying intentionality opens up the possibility of comprehending the mind

[1] See Gallagher and Varela (2003); Thompson (2007); Varela et al. (1991).

[2] See Husserl (2001), 5th Investigation, §39, and Husserl (2005). Text No. 14 (1911–1912), pp. 323–377; Marbach (2010). Marbach (2010) also notes the connection with Frege's *Begriffsschrift*.