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Peirce's cenopythagorean categories, Merleau-Ponty's chiasmatic entrelacs and Grothendieck's *Résumé*

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ABSTRACT

We present Peirce's cenopythagorean categories and Merleau-Ponty's entrelacs and chiasma, as universal phenomenological tools, particularly useful for a better understanding of dynamic, non-classical, non-separated contemporary mathematics. As a case study, we revisit Grothendieck's *Résumé*, and we explore its extremely rich mathematical, semiotical and phenomenological entanglements.

The *Résumé de la théorie métrique des produits tensoriels topologiques* (Grothendieck, 1956) can be now assessed as one of the greatest mathematical articles of the XXth century. A gigantic contribution, little understood beyond specialists, it must nevertheless reach broader audiences. Written in 1953, while Grothendieck was in his first postdoc year in Sao Paulo, received in June 1954 and published two years later in the *Boletim* of the Mathematical Society of Sao Paulo, it lay in obscurity, before it came to be recognized as anticipating by thirty years the study of the fine structure of Banach spaces (Diestel et al., 2008). Grothendieck's inventiveness in the *Résumé* goes well beyond what is usually analyzed in texts dealing with mathematical creativity, and can profit from different perspectives, both semiotical and phenomenological, which reveal better the mathematical and methodological richness of Grothendieck's achievement. On one side, Peirce's *cenopythagorean categories* (Peirce, 1886) blend freshness (*caeno*) with mathematics (*Pythagorean*), and its dense multivel superpositions and reflections can be used as a wonderful tool to unravel Grothendieck's multivel creativity. On another side, Merleau-Ponty's *entrelacs* and *chiasme* (Merleau-Ponty, 1964a, 1964b) emphasize the continuity of experience, along a dynamic web of projections and translations which help to explain Grothendieck's category-theoretic smoothness paths.

In this article we will address the question of understanding Grothendieck's *Résumé* through phenomenological perspectives, an exercise which enters into the more general realm of approaching mathematics through phenomenology (Rosen 2006). In particular, the connection between Grothendieck, Peirce and Merleau-Ponty is here observed for the first time. This yields a *natural* understanding of deep mathematical strategies (projectivity and injectivity in Hilbert and Banach spaces, unified topological-algebraic-functional analysis thought, emergence of *applied* category-theoretic methods), in which smoothness and multiplication (keys for Grothendieck) can be better revealed thanks to Peirce's *thirdness* and Merleau-Ponty's flesh and *entrelacs*. In the first section we briefly present Peirce's (phenomenological) categories, in the second section we deal with Merleau-Ponty's *entrelacs* and *chiasme*, and in the third section we apply both perspectives in order to provide an analysis of Grothendieck's *Résumé* and of the implicit emergence of mathematical categories in his work.

1. Peirce's three cenopythagorean categories

Phaneroscopy, or the study of the *phaneron*, that is the complete collective spectrum present to the mind, includes the doctrine of Peirce's cenopythagorean categories, which observe the universal modes (or "tints") occurring in phenomena. Peirce's three categories are vague, general and indeterminate, and can be found simultaneously in every phenomenon. They are intricated in several levels, but can be *prescised* (distinguished, separated, detached) following a recursive layer of interpretations, in progressively more and more determined contexts. A dialectics between the One and the Many, the universal and the particular, the general

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and the concrete is *multilayered* along a dense variety of theoretical and experimental fibers. Peirce's *Firstness* detects the immediate, the spontaneous, whatever is independent of any conception or reference to something else. *Secondness* is the category of facts, mutual oppositions, existence, actuality, material fight, action and reaction in a given world (two uses of the term "category" should not be confused in this paper: "category" alone will refer to a philosophical category, following Aristotle; "category-theoretic", "category theory", or "mathematical category" will refer to its technical mathematical sense). *Thirdness* proposes a mediation beyond clashes, a third place where the "one" and the "other" enter a dialogue, the category of sense, representation, synthesis. As (Peirce, 1886, p. 300) reckons,

> By the Third, I understand the medium which has its being or peculiarity in connecting the more absolute first and second. The end is second, the means third. A fork in the road is third, it supposes three ways. (…) The first and second are hard, absolute, and discrete, like yes and no; the perfect third is plastic, relative, and continuous. Every process, and whatever is continuous, involves thirdness. (…) Action is second, but conduct third. Law as an active force is second, but order and legislation third. Sympathy, flesh and blood, that by which I feel my neighbor's feelings, contains thirdness. Every kind of sign, representative, or deputy, everything which for any purpose stands instead of something else, whatever is helpful, or mediates between a man and his wish, is a Third.

Peirce's vague categories can be "tinctured" with keywords as follows:

(1) *Firstness*: immediacy, first impression, freshness, sensation, unary predicate, monad, chance, possibility.
(2) *Secondness*: action–reaction, effect, resistance, alterity, binary relation, dyad, fact, actuality.
(3) *Thirdness*: mediation, order, law, continuity, knowledge, ternary relation, triad, generality, necessity.

The three peircean categories interweave recursively and produce a *nested hierarchy* of interpretative modulations. A series of *modes and tones* enter the analysis, and, as we shall see, when applied to Grothendieck's *musical ear* the series helps to explain the profound complexity of the *Résumé*. The interest of Peirce's method lies in the permanent *iterative possibility* of his categorical analysis (sequences of the form *n.m.p.q* … with *n*, *m*, *p*, *q* ranging through 1, 2, 3 – see examples below in the *Résumé*). The iteration allows, in each new contextual level (*p*, *q*, …), further and further refinements of previous distinctions obtained in prior levels (*m*, *n*, …). Dynamic knowledge yields progressive *precision* through progressive *prescision*. Intelligence grows with the definition of more and more contexts of interpretation, and the association of finer and finer cenopythagorean tinctures inside each context.

The conceptual and practical back-and-forth between diverse layers is governed by the *pragmatic(ist) maxim*, which intertwines naturally with Peirce's categories. The maxim asserts that we can only attain knowledge after conceiving a wide range of representation possibilities for signs (firstness), after perusing active–reactive contrasts between sub-determinations of those signs (secondness), and after weaving recursive information between the observed semiotic processes (thirdness). The maxim acts as a *sheaf* with a double support function for the categories: a contrasting function (secondness) to obtain *local* distinctive hierarchies, and a mediating function (thirdness) to *globally* unify the different perspectives. In fact, a very broad, conceptual *differential* and *integral* calculus seems in act, to be a universal lattice of forms of reintegrating the Many into the One. It is a strategy which also recalls the basic goals of *mathematical* category theory, where the apparently different description of objects in diverse *concrete* mathematical categories is reintegrated through their universal behavior in *abstract* mathematical categories.

2. Merleau-Ponty's chiasmatic *entrelacs*

When faced with contemporary mathematics we cannot escape a certain *transitory ontology* that, at first, terminologically speaking, seems self-contradictory. Nevertheless, though the Greek *ontotetês* sends us, through Latin translations, to a supposedly atemporal "entity" or an "essence" that ontology would study, there is no reason (besides tradition) to believe that those entities or essences should be absolute and not *asymptotic*, *governed by partial gluings in a correlative evolution* between the world and knowledge. *Bimodality*, in the sense of Petitot, that is, dynamic movement both in physical and morphological-structural space, is related to such a state of things, where in fact, "things" are to be replaced by "processes" (functors, natural transformations and adjunctions in a precise category-theoretic setting). Both prefixes (*trans-*, *bi-*) provide a suitable ground to understand the wanderings of contemporary mathematics (circumscribing *modern* mathematics to the period 1830–1950, from Galois to Grothendieck, and *contemporary* mathematics to the period 1950-today).

The philosophical basis of such a dynamic ontology can be found in Merleau-Ponty's theory of *shifting*, both in the general realm of knowledge and in the particular realm of visuality, and in Badiou's specific *transitory ontology* for mathematics (Badiou, 1998) (going back to Novalis provides other forgotten foundations – see (Margantin, 1998), (Kassenbrock, 2009)). For Merleau-Ponty, the "height of reason" consists in *feeling the shifting of the soil* (Merleau-Ponty, 1964a, 92), in detecting the movement of our beliefs and supposed claims of knowing: "each creation changes, alters, clarifies, deepens, confirms, exalts, recreates or creates by anticipation all the others" (Merleau-Ponty, 1964a, 92). A complex and mobile tissue of creation surges into view, full of "detours, transgressions, slow encroachments and sudden drives", and in the contradictory coats of sediment emerges the force of creation entire. In *Eye and Mind*, Merleau-Ponty describes the body operative in the domains of knowing as a "sheaf of functions interlacing vision and movement" (Merleau-Ponty, 1964a, 16). That *sheaf* serves as an *interchange* (*à la* Serres) between the real and the imaginary, between discovery and invention, and allows us to capture the *continuous transformation of an image into its obverse*, through the various visions of interpreters. Two of the major theses of (Merleau-Ponty, 1964a) combine the necessity of both thinking the *recto/verso* dialectic and thinking in a continuous fashion:

A. What is proper to the visible is to possess a fold of invisibility, in the strict sense.

B. To unfold the world without separating thought is, precisely, modern ontology.

There is thus a compelling need to understand the obverse (Zalamea, 2013), to study knowledge without artificial divisions (yielding some of the main mathematical achievements of Grothendieck's second period, 1958–1970), and to explore the corresponding entanglement between the positive and the negative (with deep mathematical examples such as Riemann–Roch's harmony between *holomorphic* and *meromorphic* dimensions related to genera, or Peirce's harmony between *recto* and *verso* of the assertion page related to modal calculi – we will also see some wonderful harmonies between opposites in Grothendieck's