Progress in Biophysics and Molecular Biology 109 (2012) 76–94

Contents lists available at SciVerse ScienceDirect

Progress in Biophysics and Molecular Biology

journal homepage: www.elsevier.com/locate/pbiomolbio

Review

Linear and quadratic models of point process systems: Contributions of patterned input to output

K.A. Lindsay, J.R. Rosenberg*

School of Mathematics and Statistics, University Gardens, University of Glasgow, Glasgow G12 8QW, UK

ARTICLE INFO

Article history:
Available online 18 June 2012

Keywords:
Point processes
Spike trains
Volterra series
Muscle spindles
Spike rates
Spike patterns

ABSTRACT

In the 1880's Volterra characterised a nonlinear system using a functional series connecting continuous input and continuous output. Norbert Wiener, in the 1940's, circumvented problems associated with the application of Volterra series to physical problems by deriving from it a new series of terms that are mutually uncorrelated with respect to Gaussian processes. Subsequently, Brillinger, in the 1970's, introduced a point-process analogue of Volterra's series connecting point-process inputs to the instantaneous rate of point-process output. We derive here a new series from this analogue in which its terms are mutually uncorrelated with respect to Poisson processes. This new series expresses how patterned input in a spike train, represented by third-order cross-cumulants, is converted into the instantaneous rate of an output point-process. Given experimental records of suitable duration, the contribution of arbitrary patterned input to an output process can, in principle, be determined. Solutions for linear and quadratic point-process models with one and two inputs and a single output are investigated. Our theoretical results are applied to isolated muscle spindle data in which the spike trains from the primary and secondary endings from the same muscle spindle are recorded in response to stimulation of one and then two static fusimotor axons in the absence and presence of a random length change imposed on the parent muscle. For a fixed mean rate of input spikes, the analysis of the experimental data makes explicit which patterns of two input spikes contribute to an output spike.

Contents

* Corresponding author. Tel.: +44 0 141 330 5177; fax: +44 0 141 330 4111.
E-mail addresses: kenneth.lindsay@glasgow.ac.uk (K.A. Lindsay), j.rosenberg@bio.gla.ac.uk (J.R. Rosenberg).

0079-6107/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.pbiomolbio.2012.06.001

1. Introduction

The primary aim of this paper is to develop linear and quadratic models of point-process systems of the type defined in general by Brillinger (1975a), and to illustrate their application to a particular biological system (the mammalian muscle spindle). These models are analogous to the Volterra representation of nonlinear systems with continuous input and output, and were shown by Volterra to be characterisable by a series of multivariate convolution integrals (Volterra, 1959). Wiener (1958) developed a procedure for transforming the terms of the Volterra series into an equivalent series of terms that are mutually uncorrelated with respect to a white noise input.[1] The advantage of the Wiener representation of the Volterra model is that each term of the former defines an invariant property of the model so that adding further terms to the Wiener series does not change terms previously determined. By contrast, adding further terms to the Volterra representation of a system changes terms previously determined.

The development of the linear and quadratic point-process models here will show how to re-express these models in a form in which terms of its series representation are mutually uncorrelated with respect to Poisson input. This formulation of the model is the point-process equivalent of the Wiener representation of the Volterra series for a system with continuous input and output. It will be seen that the terms of this representation are invariant properties of the system in the sense that the addition of further terms to the series leaves unchanged terms previously determined. Moreover, each term of the new series has an immediate physical interpretation. In this context the work of Ogura (1972) as well as that of Hida (1970) and Krausz (1975) on orthogonal polynomials for Poisson processes is relevant.

By contrast with the non-parametric methods to be discussed in this paper, we mention another approach to the identification of point-process systems based on the use of maximum likelihood methods. In this method spike-generating models of the system are proposed and the parameters of these models are subsequently estimated from input and output data. One strategy is to model the conditional intensity function of the output process in terms of the history of the input point-processes and that of other relevant factors (Truccolo et al., 2005). Another strategy, which also models the conditional intensity function, is based on an integrate-to-threshold-and-fire model incorporating a stochastic threshold and other features that allow for spontaneous activity and a refractory period (Brillinger and Segundo, 1979; Brillinger, 1988a, b; Brillinger et al., 2009). The aim of each model is to use the history of the input processes, in combination with other factors that are thought to influence the output process, to construct the probability of an output spike for that model. From these probabilities, the probability (or likelihood) of observing the given output processes may be calculated. The aim of maximum likelihood is to find the values of the parameters of the model which maximise the likelihood that the observed output is a realisation of the model for the given input processes. Among the attractive features of the maximum likelihood method is that the statistics of the input processes are not relevant to the analysis and that these processes can be non-stationary. One objective of this approach is to assess patterns of connectivity within networks of neurons.

The plan of this article is as follows. Section 2 introduces the linear and quadratic point-process models analogous to the Volterra series representation of nonlinear systems with continuous input and output, and re-expresses these models in a form analogous to the Wiener representation of the Volterra models. Single-input single-output and two-input single-output linear and quadratic models are investigated, the former for Poisson and general input and the latter for Poisson input alone. For both the linear and quadratic models the relation between the Volterra representation and the point-process analogue of the Wiener series is made explicit. It is also shown in this section that third-order cumulants can be interpreted in terms of the traditional neurophysiological conditioning-pulse/test-pulse experimental paradigm, and that the utility of Poisson inputs is that all conditioning-pulse/test-pulse intervals are included, with the consequence that third-order interactions illustrate which patterns of input are effective in contributing to an output and which patterns are not. Section 3 presents a brief description of the experimental data used to illustrate the application of the linear and quadratic point-process models. Section 4 sets out the results in which it is demonstrated how third-order interactions contribute to second-order interactions when additional inputs are added to the system. Section 5 interprets the point-process analogue of the Wiener series for systems with continuous input and continuous output as giving a relation between the instantaneous rate of the output process and a particular combination of patterned input.

2. Point process systems

Brillinger (1975a) suggests that an important feature of point-process systems is the instantaneous rate at which the probability of an output event is generated either in the absence of input or given the history of the input process. Assuming that the point-processes are orderly, *i.e.*, at most one event can occur in any process during the interval $(t, t+\Delta t]$ for suitably small nonzero values of Δt, then in the case of a single-input single-output point-process system with $\mathcal{M}$ as input process and $\mathcal{N}$ as output process, this instantaneous rate, denoted by $\mu(t|\mathcal{M})$, is given by the value of the limit

$$\mu(t|\mathcal{M}) = \lim_{\Delta t\to 0^+} \frac{\text{Prob}\ \{\text{An}\ \mathcal{N}\ \text{event in}\ (t,\ t+\Delta t]\ |\mathcal{M}\}}{\Delta t}. \quad (1)$$

In the neurophysiological literature $\mu(t|\mathcal{M})$ is often estimated by averaging the spike train response to repeated trials using the same stimulus (*e.g.*, Marmarelis and Naka, 1974; also, see Matthews (1972, pp. 173–176) for a brief discussion of different averaging methods).

A second important consequence of orderly processes is that definition (1) has the interpretation

$$\mathbb{E}^{\mathcal{N}}[\mathrm{d}N(t)|\mathcal{M}] = \mu(t|\mathcal{M})\mathrm{d}t + o(\mathrm{d}t), \quad (2)$$

[1] For an elementary introduction to Volterra series and its relation to the Wiener theory of nonlinear systems see Marmarelis and Marmarelis (1978), and for a more advanced treatment Marmarelis (2004) or Schetzen (1980).