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Coupling multi-physics models to cardiac mechanics

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ABSTRACT

We outline and review the mathematical framework for representing mechanical deformation and contraction of the cardiac ventricles, and how this behaviour integrates with other processes crucial for understanding and modelling heart function. Building on general conservation principles of space, mass and momentum, we introduce an arbitrary Eulerian–Lagrangian framework governing the behaviour of both fluid and solid components. Exploiting the natural alignment of cardiac mechanical properties with the tissue microstructure, finite deformation measures and myocardial constitutive relations are referred to embedded structural axes. Coupling approaches for solving this large deformation mechanics framework with three dimensional fluid flow, coronary hemodynamics and electrical activation are described. We also discuss the potential of cardiac mechanics modelling for clinical applications.

Contents

1. Introduction

At its core, the human heart is a mechanical pump that drives blood flow through the cardiovascular system. The behaviour of myocardial tissue has been a major research focus for clinicians, physiologists, engineers and physical scientists who have sought to develop mathematical models to characterize cardiac mechanics (Hunter and Borg, 2003; Lee et al., 2009). A key foundation of these models is the continuum mechanical description of the heart, providing a system of partial differential equations that may be solved to simulate tissue motion and blood flow. However, the mechanical behaviour of the heart is modulated by a number of physical phenomena spanning across spatial scales, requiring the integration of continuum mechanics models with models of

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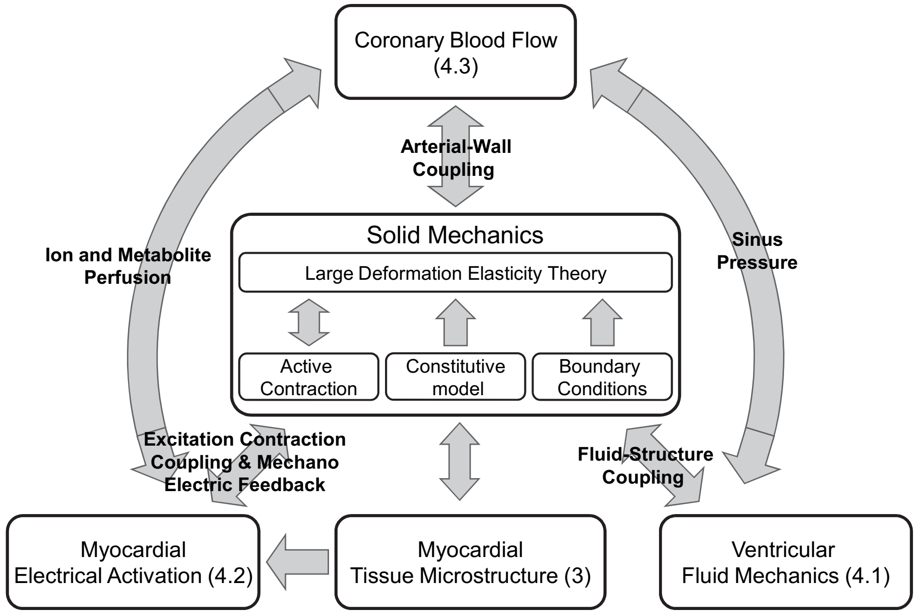

Fig. 1. The schematic relationship between mechanical components required to simulate cardiac pump function within the heart. The numbers refer to relevant sections in the text.

electrophysiology and myocardial perfusion. With improved numerical algorithms and increased computational power, the advent of more comprehensive multi-physics models has enabled novel investigations of cardiac function. In this article, we review continuum mechanical principles as they pertain to the heart as well as their interaction with other physical phenomena (as shown schematically in Fig. 1).

This paper is separated into four main parts. Section 2 provides a review of the continuum mechanical principles and equations that form the foundation for quantitative analyses of biomechanical heart function. Referring to these principles, Section 3 details the genesis of mathematical models of cardiac tissue mechanics and hemodynamics. Following this review of mechanical studies in the heart, Section 4 outlines the mathematics and physics behind the coupling of cardiac fluid/solid mechanics as well as the coupling of tissue mechanics to other physical phenomena, such as electrophysiology and coronary blood flow. Finally, Section 5 provides an overview of relevant future directions and challenges in the field of computational cardiac mechanics.

2. Continuum mechanics of blood and tissue

In this section, we provide a brief review of the equations governing tissue mechanics and blood flow, providing context for the descriptions of mechanical heart models and their coupling to additional physical phenomena (as discussed in the subsequent sections).

2.1. Coordinate frames

The continuum mechanical description of materials relies on the characterization of material motion through space and time. The variation in observed motion within a material depends strongly on how, or from what frame of reference, a material is being observed (see Fig. 2). The most fundamental is the Lagrangian reference frame, where observations are taken with respect to an initial coordinate system, denoted $\mathbf{X} = \{X_M\}$ with the gradient operator $\nabla_{\mathbf{X}} = \{\partial/\partial X_M\}$. In the Lagrangian frame, the particle time derivative, denoted D/Dt, is taken with respect to embedded coordinates, $\mathbf{X}$. This frame is useful as all information held at material points is referred back to the initial coordinate frame, providing a mechanism for observing the momentum of individual material points.

An alternative is the Eulerian frame, where observations are taken with respect to a static spatial coordinate system, denoted $\boldsymbol{x} = \{x_i\}$ with the gradient operator $\nabla_x = \{\partial/\partial x_i\}$. In the Eulerian frame, the particle time derivative, denoted $\partial/\partial t$, is taken with respect to fixed spatial coordinates, $\boldsymbol{x}$. As a result, observations of a material are observed through time in the material's current configuration as it passes through fixed observation points in space.

Both Lagrangian and Eulerian frames can be generalized as special cases of the so-called Arbitrary Lagrangian–Eulerian (ALE) frame, where observations are taken with respect to an arbitrary moving coordinate system, denoted $\boldsymbol{\eta} = \{\eta_i\}$ with the gradient operator $\nabla_{\boldsymbol{\eta}} = \{\partial/\partial \eta_i\}$. The particle ALE time derivative, denoted ∂_t is taken with respect to a fixed coordinate, $\boldsymbol{\eta}$, which may appear as moving points in the Eulerian or Lagrangian frames. Choosing the motion of the ALE frame to be fixed in space or to convect with the material, the Eulerian or Lagrangian frames may be observed.

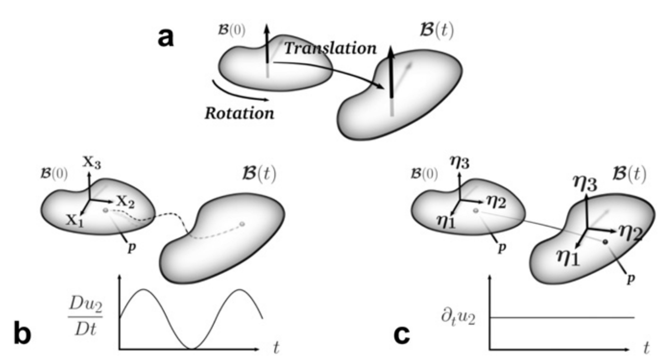

Fig. 2. a) Example domain undergoing rotation about a central axis and translation. b) A particle point, p, in the Lagrangian frame is shown with its position in the body. The Lagrangian time derivative appears sinusoidal (due to the rotation), and offset from zero (due to the translation observed). c) A fixed point, p, in the ALE frame (which translates, but does not rotate with the body) is shown. The ALE time derivative appears constant (due to rotation and translation) as the material moves through the observation point, p.