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Quantum cognition based on an ambiguous representation derived from a rough set approximation

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ABSTRACT

Over the last years, in a series papers by Arecchi and others, a model for the cognitive processes involved in decision making has been proposed and investigated. The key element of this model is the expression of apprehension and judgment, basic cognitive process of decision making, as an inverse Bayes inference classifying the information content of neuron spike trains. It has been shown that for successive plural stimuli this inference, equipped with basic non-algorithmic jumps, is affected by quantum-like characteristics. We show here that such a decision making process is related consistently with an ambiguous representation by an observer within a universe of discourse. In our work the ambiguous representation of an object or a stimuli is defined as a pair of maps from objects of a set to their representations, where these two maps are interrelated in a particular structure. The a priori and a posteriori hypotheses in Bayes inference are replaced by the upper and lower approximations, correspondingly, for the initial data sets that are derived with respect to each map. Upper and lower approximations herein are defined in the context of "rough set" analysis. The inverse Bayes inference is implemented by the lower approximations with respect to the one map and for the upper approximation with respect to the other map for a given data set. We show further that, due to the particular structural relation between the two maps, the logical structure of such combined approximations can only be expressed as an orthomodular lattice and therefore can be represented by a quantum rather than a Boolean logic. To our knowledge, this is the first investigation aiming to reveal the concrete logic structure of inverse Bayes inference in cognitive processes.

1. Introduction

Cognition in the process of decision making has two distinct features, namely *apprehension* and *judgment*. By apprehension we understand the coherent perception correlated with the recruitment and engagement in collective behavior of the required neuronal populations and by judgment we understand the process of comparing and of correlating – at least two – earlier apprehensions. Judgment requires memory and a certain degree of self-awareness, in the sense that the observer can hold on and internalize as its own a set of previously apprehended stimuli. Both processes require some kind of representation which entails their conceptualization; evidently both processes afford context dependencies within a certain framework of cognitive references. Arecchi critically discuss this issue in (Nicolis and Basios, 2015; Arecchi, 2007a,b; Arecchi et al., 2012) and points out its quantum-like nature.

Recently theoretical and experimental advances in the area of quantum cognition and decision highlight this quantum-likeness of conceptualization. This endeavor brings together results from cognitive mathematical psychology (Pothos and Busemeyer, 2013), 'rational decision theory' and finance (Pothos and Busemeyer, 2009; Segal and Segal, 1998; Baaquie, 2004), experimental and theoretical artificial intelligence (Aerts and Czachor, 2004; Gabora and Aerts, 2002), and quantum probability's logical structure (Piron, 1976; Mackey, 1963; Aerts, 1999). Much is owned to the seminal work of Aerts from 1995 till now (Aerts and Aerts, 1995; Aerts, 2009, 2010; Aerts et al., 2013a,b; Aerts and Gabora, 2005a,b; Aerts and Sozzo, 2011; Aerts et al., 2014; Aerts and de Biachi, 2015). Nowadays, quantum-like behavior in language, concept correspondence and operational research and decision making have been developed in several publications by a considerable community of authors.

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For a comprehensive overview of the field one can consult the reference works covered in the recently published books by Busemeyer and Bruza (2012), and by Khrennikov (2010).

These authors focus on the probabilistic aspects and statistical verifications of structural aspects of quantum logic and quantum probability, while the studies of the dynamical basis of cognitive processes involved in decision making usually follow a more microscopic classical approach based on neural dynamics, see for example (Busemeyer and Bruza, 2012; Wojcik et al., 2011). Notably the role of chaotic dynamics in biological information processing, pioneered by the work of Nicolis and Tsuda (1985) has been theoretically and experimentally investigated over the last decades by the group of Walter Freeman, K. Kaneko and others (an overview in Nicolis and Basios (2015), Nicolis and Tsuda (1985), particularly chapters 13, 15, 16, 17). Arecchi's proposal aims at bridging these two aspects, i.e. structural and dynamical, by constructing a model based on data sets from spike train synchronization and recursive (forward and inverse) Bayesian inference. The information content of the data sets of recorded neural spike-trains and analyzed by entropic measures enabling thus the tracing of suitable cross- and auto-correlation indexes. Based on such an approach Arecchi and his team conjecture and offer estimations for the existence of a quantal constant (Note: This is not the standard Plank constant of quantum physics but rather a quantum-like constant in the spirit of quantal neuron discharge proposed by Katz (1971)) underlying specific quantum-like information processing for visual stimuli. As Arecchi (2003, 2004, 2005) demonstrates the operation of the observing apparatus is equivalent to the operation of monitoring a Wigner distribution. The reported time coding implies a Wigner non-local measurement that stores and reads data in a global rather than a serial local fashion.

In the present work we built further on the procedure proposed by Arecchi and his team by considering an ambiguous representation based on "rough set analysis (Pawlak, 1981, 1982; Polkowski, 2002; Doherty et al., 2006; Gunji et al., 2009; Haruna and Gunji, 2009; Gunji and Haruna, 2010)" via a pair of mappings that serve as a model for Bayesian recurrent inferences within the conceptual categorization of stimuli afforded by an observer in a universe of discourse. We show that this can account, in a consistent way, for the quantum theoretical conjecture of Arecchi. We are able to show this by examining the underlying logic of the pair of mappings which turns out to be an orthomodular lattice as the ones related to quantum logic (Kalbach, 1983; Svozil, 1993, 1998).

The idea of a pair of Bayes and inverse Bayes inference could be applied not only to cognitive but also general biological systems. Since Bayes inference is based on replacing the probability with the conditional probability and inverse Bayes inference is based on replacing the conditional probability with the probability, they can constitute the interpolating system of the ordered structure consisting of parts and whole. The more conditions are added, the lower the probability is located. Although the ordered relation is defined well, the structure is mixed with up and interpolated with each other. In other words, there is a strict hierarchical structure on one hand, and different layers of the hierarchy are interpolated with each other to destroy the hierarchical structure on the other hand. That is a kind of dynamical hierarchical structure.

We here show that the interaction of Bayes and inverse Bayes inference could lead to the distributed system in which each subsystem is independent of other subsystems to some extent and has attribute to its own specialized role. Although in each subsystem any external stimulus is reduced to atomistic elements, external stimulus taken in multiple subsystems cannot be reduced to atomistic elements. Thus a system is partly regarded as a reductionism, but is actually non-reductionism. Our result suggests that the complex making decision could result in dynamic hierarchical structure.

The paper is organized as follows, in Section 2 we briefly present the approach outlined by Arecchi et al. and discuss the inverse Bayesian procedure and its non-algorithmic aspects. Section 3 is the main exposition of our construction. In its first sub-section we develop the basic rationale for our rough set analysis framework. In the second sub-section we construct the two maps that implement an ambiguous representation based on the previous analysis and we offer specific examples for our model. In the third sub-section we discuss the derivation of a lattice of logical propositions from a rough set approximation. The fourth, and closing sub-section, of the third section presents the proof that the derived lattice of logical propositions coming from the above ambiguous representation is orthomodular, hence related to a quantum-logic lattice rather than a classical Boolean one. In Section 4 we revisit Arecchi's quantum-like hypothesis and we show that our scheme is consistent with his conjecture. Concluding in Section 5, we discuss the prospects of rough set approximation as a model for the inverse Bayes inference process and their non-algorithmic aspects and the outlook that this might entail for further investigations.

2. From apprehension to judgment and quantum Bayes inference

In this section we briefly present the quantum theoretical conjecture proposed by Arecchi. As discussed in Arecchi (2011) *apprehension* consists of a coherent perception which emerges from the recruitment of neuronal groups, and *judgment* consists by recalling in memory previous apprehended units, coded in a suitable language, which subsequently are compared and selected in order to attain a judgment.

The first process, apprehension, has a duration around 1 s; its associated neuronal correlate can be traced by EEG (electroencephalogram) signals synchronized in the "gamma band" (frequencies between 40 and 60 Hz) coming from distant cortical areas. It can be described as an interpretation of the sensory stimuli on the basis of available algorithms, through a (forward) Bayes inference.

Following (Arecchi, 2007a,b, 2011), let h be the interpretative, competing, hypotheses in the presence of a sensory stimulus piece of data, d, the (forward) Bayes inference, selects the most plausible hypothesis, h^*, that determines the motor reaction, exploiting an a priori existing algorithm, $P(d|h)$. That represents the conditional probability that a datum d is conforming with an hypothesis h. The $P(d|h)$ are given e.g. have been learned during the past. They represent the equipment, or faculties, by which a cognitive agent, be it natural or artificial, faces and interprets its universe of discourse. For example, equipping a robot with a convenient set of $P(d|h)$, we expect a sensible behavior within a given environment providing sensory input translated to its repertoire of responses.

The second process, judgment, entails a comparison between two apprehensions acquired at different times, coded in a given language and recalled by memory. Let us call d the code of the second apprehension and h^* the code of the first one, now – at variance with apprehension – it is h^* that is already given; and instead the relation $P(d|h)$ which connects them must be retrieved, it represents the conformity between d and h^*, that is, the best interpretation of d in the light of h^*. For example, as in Arecchi (2007a,b, 2011), two successive pieces of the text can be compared and the conformity of the second one with respect to the first one can be determined. This is very different from apprehension, where there is no problem of conformity but of plausibility of h^* in view of a motor reaction. In Arecchi (2007a,b, 2011) the following two examples serve as illustration: "a rabbit perceives a rustle behind a hedge and it runs away, without investigating whether it was a fox or just a blow of wind. . . . As Freeman (2001) puts it the life of a brainy animal