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An emergence of formal logic induced by an internal agent

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ABSTRACT

In this paper, we mainly address three issues: externality of an agent, purpose of an agent, and a kind of "softness" of components in a system. Agents are independent of a system in an ordinary multi-agent model, hence the behavior of a system is not autonomous but influenced by the agents. If a multi-agent model is considered as a completely autonomous one, agents in the model are inevitably deprived of their externality and independence from the model. In order to treat the completely autonomous transition of a system, we introduce an agent which is part of a system, and has a purpose which is independent from the system. The interaction between a system and an agent transforms a random graph corresponding to the system into the graph which represents formal logic adequately. In the emergent graph, there are many complete subgraphs, which can be regarded as conceptualized matters. We modify the definition of a conceptualized matter into a subgraph which is a cycle of arrows, and regard the density of arrows of each conceptualized matter as validness. We define this object with the density as a soft object. A complete graph has a maximum number of arrows, hence is the most reliable soft object. In a similar way, we call an arrow with validness a soft arrow, and treat the relation between soft objects and soft arrows. The argument of this paper is relevant to dynamical formal logic, and at the same time, is intended to serve as a basis for an agent model.

1. Introduction

We propose a novel model of dynamical formal logic, especially which elaborates the emergence of formal logic. Dynamical transition of formal logic was dealt with by Gunji et al. (2004) in the context of informorphism by Barwise and Seligman (1997). Gunji et al. (2006) also proposed another model based on lattice theory (Davey and Priestley, 2002). We also already presented this in (2007, 2008) in the form of a multi-agent model (Wooldridge, 2009). However in this paper, we raise a problem with a multi-agent model as below.

A multi-agent model assumes at least one agent by definition. What is an agent? To answer this question, firstly we presume that an agent is what is simply transformed in a system. If agents of a system are completely independent of, and external to the system, the behavior of the system can be attributed to the behaviors of agents. Thus, we must check up on the property of agents in order to argue for the property of a multi-agent model. This may lead to infinite regress. Responding to this situation, instead of external agents, we introduce an agent which exists inside a system, in other words, which is a part of a system. The model which we propose is an internal measurement model of formal logic, where internal measurement was proposed by Matsuno (1989). We call an agent which is inside a system completely an internal agent, and also call the model Internal Agent Model.

Another major characteristic of an agent is its autonomy. We define a guiding principle which is inherent in each agent and leads to the autonomous ability, and call it a "purpose". Thus, an agent in Internal Agent Model has two main characteristics: internal and autonomous.

Classical propositional logic can be composed only of negation and implication. Here, we treat only implication represented by a directed graph (Harary, 1969). It is sufficient because one directed edge between two nodes represents an implicational relation between them, and the absence of a directed edge represents the negation of the implicational relation. However, not every directed graph adequately represents formal logic. We observe the emergence of a directed graph which represents formal logic by the action of an internal agent. Gunji and Higashi (2001) also argued exactly about the relation between directed graphs and category theory (Mac Lane, 1998).

We here make the purpose of an internal agent as the origination of the transitive law of implication. Ordinarily the fundamental property of a logical system is given in the form of an

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axiom dogmatically, and the same applies to the transitive law of implication. Instead of this situation, in this paper, we introduce the transitive law into the formal system as the purpose of an internal agent. This kind of introduction means differentiation or localization of the axiom. In a formal system in which a law stands simultaneously throughout the whole system, the temporal consideration of the law is ignored. In addition, this introduction enables the system to transform itself continuously, in contrast with the ordinary axiomatic systems which vary discontinuously according to which axioms are adopted.

In addition, we also argue for logical objects in the process of observing the transition of a directed graph. The object in formal logic is obvious. For example, it has the property of the reflexive law: X is X. In contrast with the obviousness, there is a critical problem such as Russell's paradox (Whitehead and Russell, 1925). We present an attempt to solve this problem by introducing the notion "softness" into logical objects.

While we regard the system as a mere graph out of context, the internal agent is nothing more than a subgraph. That is to say, the interaction between the system and the internal agent which we propose in the paper is the interaction between a graph and its subgraph. Moreover, from the definition of the purpose of the internal agent, we can regard the model as the independent applications of the transitive law to either the whole or the part of a system. In a similar way, the notion of softness of an object leads to the uncertainty of the reflexive law (the obviousness of the object). In short, we aim to observe the dynamical feature of formal logic in which the fundamental laws are either deprived or partially adopted.

The paper is organized as follows: firstly we define an internal agent inside a system. An internal agent differs from a part of the system only in that it has a purpose, that is, an internal agent is nothing more than a mere part of the system which has a purpose. Next, we schematize the purpose of an internal agent, and define the interaction between a system and an internal agent. In Section 3, we observe the emergence of a directed graph which represents formal logic adequately out of the interaction, and look into the results under some various conditions. We also observe some distinctive features of the emergent graph. In order to elaborate these features, we define the notion of softness of both an object and an arrow in Section 4. And then we check some results from particular cases in order to discuss the softness of object and arrow, especially the influence of soft arrows on soft objects. Finally, we sum up the difference of tendency among the values of some parameters, however in any case, all the emergent graphs can be regarded as formal logic.

2. Internal Agent Model

2.1. System and internal agent

Hereafter, we treat only the implicational fragment of propositional logic as mentioned in the preceding section. We are concerned only with a directed graph (Harary, 1969), which can represent naturally a set of implications. We represent a system composed of objects and arrows between objects by a directed graph. Naturally, an object and an arrow correspond to a node and a directed edge in a directed graph, respectively. There is no arrow from an object to itself, and there is at most one arrow between an ordered pair of objects. These settings are for convenience of explanation.

In an ordinary multi-agent model, an agent exists independently outside of the world which is represented by the whole of the system. That is, the agent is an observer and the world is the observable. There is a rigid distinction between them. However, we consider that the externality of an agent is a mere postulate. The agent obviously requires matters of the world, which it thinks about or treats. The knowledge which the agent has consists of the components of the world, hence we can regard an aggregate of the components as an agent itself. Thus, we set out an agent inside the world. For instance, when the world is represented by a directed graph, we regard a particular subgraph as an agent. Fig. 1 shows an example. Due to this setting, we can treat an agent and objects which are observable things of the world on the same level. We prefer not to discriminate an agent from a system in order to describe completely independent transitions of a system. In addition, an agent becomes nothing more than an object which can be observed from the standpoint of internal measurement (Matsuno, 1989). An agent as an object can be naturally influenced by a system. Therefore, there may be interaction between an agent and a system. Now we call such part of a system an internal agent. We sometimes refer to internal agent as agent hereafter.

Another main characteristic of an agent is its autonomy. In general, agents are treated as if agents act autonomously in a system. The autonomous behavior of an agent requires a guiding principle which is inherent in the agent and can vary according to circumstances, though it may not be seen. We call the guiding principle a purpose. The system which is the outside of the agent cannot affect the purpose of the agent by definition. Indeed we give a purpose to a part of a system and regard the part as an internal agent, and the purpose is independent of the system.

Based on the above understanding, in this paper we define an internal agent as an object of a world which has purpose.

2.2. Purpose of agent

An arbitrary directed graph does not necessarily hold all the properties of formal logic. Hence not every directed graph represents adequately formal logic. We pay notice to the transitive law in this paper as well as our previous papers (Sawa and Gunji, 2007, 2008). We define an index to show the emergence of the transitive law, as follows.

Definition 1. (*Transitivity rate*). Given a directed graph G, TR is defined as

$$\mathrm{TR} := |G|/|G'|,$$

where $|G|$ is the number of directed edges in G, and G' is the graph transformed from G, in which the transitive law holds completely by adding a minimum number of requisite directed edges.

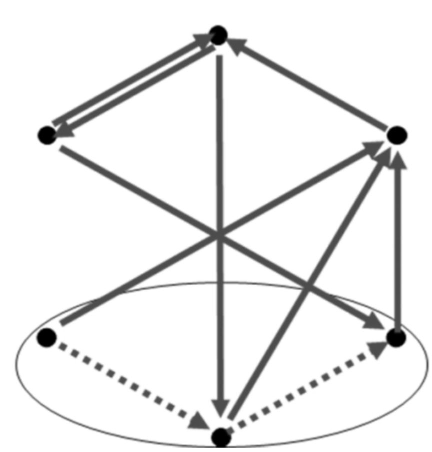

Fig. 1. An example of a system and an internal agent. While the whole directed graph represents a system, an internal agent is the part of the graph represented by dashed arrows.