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Research Article

Interactions between species and environments from incomplete information

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ABSTRACT

There are two contradictory aspects of the adaptive process in evolution. The first is that species must optimally increase their own fitness in a given environment. The second is that species must maintain their variation to be ready to respond to changing environments. In a strict sense, these two aspects might consider to be mutually exclusive. If species are optimally adapted, then the variation in the species that is suboptimal decreases and vice versa. To resolve this dilemma, species must find a balance between optimal adaptation and robust adaptation. Finding the balance between these processes requires both the local and global complete, static information. However, the balance between the processes must be dynamic. In this study, we propose a model that illustrates dynamic negotiation between the global and local information using lattice theory. The dynamic negotiation between these two levels results in an overestimate of fitness for each species. The overestimation of fitness in our model represents the multiplicity of fitness which is sometimes discussed as the exaptation. We show that species in our model demonstrate the power law of the lifespan distribution and $1/f$ fluctuation for the adaptive process. Our model allows for a balance between optimal adaptation and robust adaptation without any arbitrary parameters.

1. Introduction

Adaptation is one of the intriguing issues for vast biological events such as evolution and ecology (Darwin, 1859; MacColl, 2011). Each species must survive in varied environments. Generally, we consider fitness as explaining how species adapt to an environment. "Survival of the fittest", proposed by Darwin (Darwin, 1859), is the most famous theory for adaptation and evolution. Many models for evolution have therefore applied the concept of fitness for better or worse (Bak and Sneppen, 1993; MacColl, 2011; Mustonen and Lässig, 2010; Sneppen et al., 1995). The celebrated model of Bak and Sneppen explained biological evolution using a fitness landscape (Bak and Sneppen, 1993; Sneppen et al., 1995). Their model is very simply constructed. Each species has its own fitness that is independent of other species, and species connect to each other in a flood chain. A selected less fit species changes its fitness randomly. The neighboring species in the food chain then also changes its value of fitness at random. Bak and Sneppen explained the biological evolution of an ecosystem; for example, the power laws for extinction's distribution, using this simple model (Bak and Sneppen, 1993; Sneppen et al., 1995).

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The evolutional model of Bak and Sneppen provides many suggestions for considering a species' adaptation in the fitness landscape. The problem with their model, which we address here, is the assumption of perfect information on fitness because the model must know all of the fitness values to decide the least fit. The requirement for knowing all of the precise fitness values assumption seems unnatural. From Darwin's evolutional theory, we surely see the results of a species' fitness. However, we often observe that nature permits unnecessary traits in species during their evolution. The concept of exaptation, proposed by Gould (Gould, 1999; Buss et al., 1998), could explain the existence of these traits, which seem to be unnecessary. Exaptation has two meanings (Gould, 1999). One is "a feature, now useful to an organism, that did not arise as an adaptation for its present role, but was subsequently co-opted for its current function". The other is "a feature that now enhance fitness, but were not built by natural selection for their current role". A famous example of exaptation is the wings of birds. It is believed that feathers were originally used for thermal regulation before they were used for flight. This example suggests that the traits of animals would potentially have various possible functions that could be adapted to many situations. Therefore, we believe that exaptation would permit multiplicity of the fitness in nature because of the way that trait usage is open in an environment. Fitness, which is imposed by the environment, is not determined definitively as in the model of Bak and Sneppen. In the living world, it is very hard to obtain precise information, such as absolutely knowing all fitness levels.

The assumption of complete precise information of fitness would be a risk for erasing the essential problem of adapting species to their environment. One typical problem is finding the balance between optimal adaptation and robust adaptation (Eigen et al., 1989; Wilke, 2005). The robust adaptation means that the system must keep high diversity of species to avoid whole extinction. The optimal adaptation means that the system must contain as many species with the highest fitness as possible to increase their whole fitness. These two adaptive processes, however, are mutually exclusive. If we start from complete precise information of fitness, we do find the optimal balance between two adaptive processes because the definition of fitness is utterly imposed on the modeler (Jansen and Stumpf, 2005; Lande and Shannon, 1996). If nature finds the balance between them by itself, it is important to consider what is incomplete information of fitness when we construct a model for the adaptation.

The incomplete information of fitness would be considered as the multiplicity of fitness as we discussed the example of exaptation. Multiplicity would provide a certain interval for the selection among species' traits when they decide the necessary–unnecessary traits for their survival. Of course, the reference point of the selection among species' traits strongly depends on the context where they live because the usage of traits also depends on their environment. On the other hand, each individual measures its fitness from its local incomplete observation to adapt to the environment. Therefore, the implementation of the multiplicity of fitness is accomplished by reflecting both aspects, which are the local conditions and the global (or their context) conditions.

In this paper we use lattice theory to represent the interface between the local and the global fitness in the adaptive process to their environment. Each species observes only parts of the environment. This incomplete local fitness is represented as fitness blocks. The fitness blocks represent the necessary–unnecessary portion in their traits by using binary bit strings. We construct a lattice from these fitness blocks. The lattice structure ensures the order relation among each fitness block and avoids the existence of non-comparable fitness block (element) with all other elements. The lattice structure also evolves with time steps because the relation among fitness blocks dynamically changes. In our evolving lattice model, local incomplete information drives the evolution of the constructed lattice. The global context of the fitness in our model represents as the congruence of the lattice. To use the congruence, the system re-estimates the incomplete local fitness using a quotient lattice. The quotient lattice plays a role in identifying a set of fitness blocks as representative one fitness blocks. We show that species in our model can obtain robust adaptation without any parameter tuning using a quotient lattice.

2. Basic concepts

To construct our model, we use a lattice theory. The lattice theory has widely used in the computer science such as an automaton theory (Davey and Priestelely, 2005; Gunji et al., 2006; Vecchio et al., 2006). Here we review the basic definitions and notion, which are used in our model for unfamiliar readers of the lattice theory.

Definition 2.1 *((Partial order))*. Let P be a set. An order on P is a binary relation $\leq$ on P such that, for all $x, y, z \in P$

(i) $x \leq x$
(ii) $x \leq y$ and $y \leq x \Rightarrow x = y$
(iii) $x \leq y$ and $y \leq z \Rightarrow x \leq z$□

We denote a partially ordered set by the pair $(P, \leq)$. For example, a set of bit (binary) strings can construct a partial order. A bit string $a_1a_2a_3 \cdots a_n$ is a finite sequence of zero or one ($a_i \in \{0, 1\}$). An order between two bit strings such as $a_1a_2a_3 \cdots a_n$ and $b_1b_2b_3 \cdots b_n$ is defined by $a_1a_2a_3 \cdots a_n \leq b_1b_2b_3 \cdots b_n$ if $a_i \leq b_i$ for all i. We use a set of bit strings in this study. However, A partial order is not a lattice. Then we define the *meet* and the *join*. We define the join "$\vee$" and the meet "$\wedge$" of two elements x and y in P. The join can be defined by $x \vee y = sup\{x, y\}$ when it exists. The join can be defined by $x \wedge y = inf\{x, y\}$ when it exists. The notation of sup (inf) means the lowest (greatest) upper bound of $\{x, y\}$ in P.

Definition 2.2 *((Lattice))*. Let $(P, \leq)$ be a non-empty partially ordered set.

If $x \vee y$ and $x \wedge y$ exist for all $x, y \in P$, then $(P, \leq)$ is called for a lattice.□

To distinct a partially ordered set, we denote a lattice as $(L, \leq, \wedge, \vee)$. In this paper, there are often-used sets that are an ideal and a filter. An ideal is used when we construct the congruence on a lattice.

Definition 2.3 *((Ideal))*. Let $(L, \leq, \wedge, \vee)$ be a lattice. A non-empty subset of J is called an *ideal* if

(i) $x, y \in J$ implies $x \vee y \in J$.
(ii) $x \in L$, $y \in J$ and $x \leq y$ imply $x \in J$.□

Definition 2.4 *((Filter))*. Let $(L, \leq, \wedge, \vee)$ be a lattice. A non-empty subset of F is called an *filter* if

(1) $x, y \in F$ implies $x \wedge y \in F$.
(2) $x \in L$, $y \in F$ and $y \leq x$ imply $x \in F$.□

The typical example of an ideal is a down set on a lattice. The definition of a down set is a subset $J = \{y \in L | y \leq x\}$ when $x \in L$. We denote a down set of x as $\downarrow x$. It can easily be rerified that a down set satisfies the condition of ideal. In a similar way, we can define an upper set on a lattice such as $F = \{y \in L | x \leq y\}$ when $x \in L$. We denote an upper set of x as $\uparrow x$. We can also rerify that an upper set satisfies the condition of a filter. Next, we consider congruence on a lattice to define a quotient lattice. A congruence is an equivalence relation which is restricted by a certain condition.

Definition 2.5 *((Congruence on a lattice))*. Let $(L, \leq, \wedge, \vee)$ be a lattice. Let an equivalence relation on L be $\theta = \{\langle x, y \rangle \in L \times L\}$ such that any $x, y, z \in L$,

(i) $\langle x, y \rangle \in \theta$.
(ii) $\langle x, y \rangle \in \theta \Leftrightarrow \langle y, x \rangle \in \theta$.
(iii) $\langle x, y \rangle \in \theta$ and $\langle y, z \rangle \in \theta \Rightarrow \langle x, z \rangle \in \theta$.

We also denote $\langle x, y \rangle \in \theta$ as $x \equiv y \pmod \theta$. Then an equivalence relation is a congruence on L, if for any $x, y, z, w \in L$, ($x \equiv y \pmod \theta$ and $z \equiv w \pmod \theta$) $\Rightarrow$ ($x \vee z \equiv y \vee w \pmod \theta$ and $x \wedge z \equiv y \wedge w \pmod \theta$).□

Then we can make a quotient lattice by using a congruence.

Definition 2.6 *((Quotient lattice))*. Let θ be a congruence on a lattice $(L, \leq, \wedge, \vee)$, then a set L/θ is defined by

$$L/\theta = \{[x]_\theta | x \in L\} \quad \text{with} \quad [x]_\theta = \{y \in L | x \equiv y \pmod \theta\}$$

The join and the meet on L/θ are defined by

$$[x]_\theta \wedge [y]_\theta := [x \wedge y]_\theta, \; [x]_\theta \vee [y]_\theta := [x \vee y]_\theta$$

Then we call $(L/\theta, \leq, \wedge, \vee)$ the quotient lattice of L modulo θ.□

In this study, we construct a quotient lattice from a given ideal. First we introduce the equivalence relation derived from an ideal.