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Species extinction and permanence of an impulsively controlled two-prey one-predator system with seasonal effects

Hunki Baek

Department of Mathematics, Kyungpook National University, Sankyeuk Dong, Daegu 702-701, South Korea

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ABSTRACT

Recently, the population dynamic systems with impulsive controls have been researched by many authors. However, most of them are reluctant to study the seasonal effects on prey. Thus, in this paper, an impulsively controlled two-prey one-predator system with the Beddington–DeAngelis type functional response and seasonal effects is investigated. By using the Floquet theory, the sufficient conditions for the existence of a globally asymptotically stable two-prey-free periodic solution are established. Further, it is proven that this system is permanent under some conditions via a comparison method involving multiple Lyapunov functions and meanwhile the conditions for extinction of one of the two prey and permanence of the remaining two species are given.

1. Introduction

The dynamical relationships between predator and prey can be represented by the functional response which refers to the change in the density of prey attached per unit time per predator as the prey density changes. One of well-known functional responses is the Beddington–DeAngelis functional response introduced by Beddington (1975) and DeAngelis et al. (1975), independently. The main difference of this functional response from a classical Holling type one is that this one contains an extra term presenting mutual interference by predators. There are a lot of factors to be considered in the environment to describe more realistic relationships between predator and prey. One important factor is seasonality, which is a kind of periodic fluctuation varying with changing seasons. Also, the seasonality has an effect on various parameters in the ecological systems. For this reason, it is valuable to carry out research on systems with periodic ecological parameters which might be quite naturally exposed, such as those due to seasonal effects of weather or food supply, etc. (Cushing, 1977; Sabin and Summers, 1993). There are several ways to reflect the effects caused by the seasonality on ecological systems (Liu et al., 2005, 2006; W.B. Wang et al., 2007). In this paper we consider the intrinsic growth rate a of the prey population as a periodically varying function of time due to seasonal variations. In other words we adopt $a_0 = a(1 + \epsilon\ \sin(\omega t))$ as the intrinsic growth rate of the prey. Here the parameter ϵ represents the degree of seasonality, $a\epsilon$ the magnitude of the perturbation in a_0 and ω the angular frequency of the fluctuation caused by seasonality.

Moreover, there are still some other factors that affect the ecological system, such as fire, flood, harvesting seasons, etc., that are not suitable to be considered continually. These impulsive perturbations bring sudden change to the system. Such impulsive systems are found in almost every area of applied science and have been examined in a variety of studies: impulsive birth (Roberts and Kao, 1998; Tang and Chen, 2002), impulsive vaccination (Donofrio, 2002; Shulgin et al., 1998), chemotherapeutic treatment of disease (Lakmeche and Arino, 2000; Panetta, 1996), impulsive releasing of infective pests (Georgescu and Zhang, 2008), optimizing problems of an impulsive system with augmentative biological control (Mailleret and Grognard, 2008), and so on. In particular, the impulsively controlled prey–predator population systems have been investigated by a number of researchers (Baek, 2009; Georgescu and Morosanu, 2008; Liu et al., 2005, 2006; Liu and Chen, 2003; Song and Xiang, 2006; Tang et al., 2005; W. Wang et al., 2007; Wang et al., 2008; Zhang and Chen, 2005a, b, 2006;

E-mail address: hkbaek@knu.ac.kr.

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Zhang et al., 2005a, b, 2006a, b; Y. Zhang et al., 2003, 2005). Thus the field of research of impulsive differential equations with impulsive control terms seems to be a new growing area of interest in recent years. However, the two-prey and one-predator systems with seasonal effects and impulsive controls are less noticeable in spite of their importance. Now we develop the following new system with seasonality by bringing in a proportional periodic impulsive harvesting such as spraying pesticide for all species and a constant periodic releasing for the predator at different fixed times.

$$\begin{cases} \left.\begin{array}{l} x_1'(t) = x_1(t)\left(a_1 + \gamma_1\,\sin(\theta_1 t) - b_1x_1(t) - c_1x_2(t) - \dfrac{\sigma_1 y(t)}{1+d_1x_1(t)+e_1x_2(t)+\mu_1 y(t)}\right), \\ x_2'(t) = x_2(t)\left(a_2 + \gamma_2\,\sin(\theta_2 t) - b_2x_2(t) - c_2x_1(t) - \dfrac{\sigma_2 y(t)}{1+d_2x_1(t)+e_2x_2(t)+\mu_2 y(t)}\right), \\ y'(t) = y(t)\left(-a_3 + \dfrac{\sigma_3 x_1(t)}{1+d_1x_1(t)+e_1x_2(t)+\mu_1 y(t)} + \dfrac{\sigma_4 x_2(t)}{1+d_2x_1(t)+e_2x_2(t)+\mu_2 y(t)}\right), \end{array}\right\} \\ t \neq (n+\tau-1)T, \qquad t \neq nT, \\ \left.\begin{array}{l} x_1(t^+) = (1-p_1)x_1(t), \\ x_2(t^+) = (1-p_2)x_2(t), \\ y(t^+) = (1-p_3)y(t), \end{array}\right\} \quad t = (n+\tau-1)T, \\ \left.\begin{array}{l} x_1(t^+) = x_1(t), \\ x_2(t^+) = x_2(t), \\ y(t^+) = y(t)+q, \end{array}\right\} \quad t = nT, \\ (x_1(0^+), x_2(0^+), y(0^+)) = (x_{01}, x_{02}, y_0), \end{cases} \tag{1}$$

where $x_1(t)$, $x_2(t)$ and $y(t)$ represent the population densities of the two prey and the predator at time t, respectively. The constant $a_i (i = 1, 2)$ are called the intrinsic growth rates of the prey population, $b_i (i = 1, 2)$ are the coefficients of intra-specific competition, $c_i (i = 1, 2)$ are the parameters representing competitive effects between the two prey, $\sigma_i (i = 1, 2)$ are the per-capita rates of the predation of the predator, $d_i (i = 1, 2)$ and $e_i (i = 1, 2)$ are the half-saturation constants, the constant a_3 is the death rate of the predator, the terms $\mu_i (i = 1, 2)$ scale the impact of the predator interference, $\sigma_i (i = 3, 4)$ are the rates of the conversing prey into the predator, λ and ω represent the magnitude and the frequency of the seasonal forcing terms, respectively, T is the period of the impulsive immigration or of the stock of the predator, $0 < \tau < 1$, $0 \leq p_1, p_2, p_3 < 1$ present the fraction of the two prey and the predator which die due to harvesting or pesticides at $t = (n+\tau-1)T$ and q is the size of immigration or of the stock of the predator.

The main purpose of this paper is to investigate the conditions for the extinction of the two-prey and for the permanence of the system (1). In addition, we provide some numerical simulations to substantiate our theoretical results.

Observe that if $\gamma_1 = \gamma_2 = c_1 = c_2 = 0$, $d_1 = d_2 > 0$, $e_1 = e_2 > 0$ and $\mu_1 = \mu_2 > 0$, the system (1) becomes a special Beddington-type system with impulsive effects as follows:

$$\begin{cases} \left.\begin{array}{l} x_1'(t) = x_1(t)\left(a_1 - b_1x_1(t) - \dfrac{\sigma_1 y(t)}{1+d_1x_1(t)+e_1x_2(t)+\mu_1 y(t)}\right), \\ x_2'(t) = x_2(t)\left(a_2 - b_2x_2(t) - \dfrac{\sigma_2 y(t)}{1+d_1x_1(t)+e_1x_2(t)+\mu_1 y(t)}\right), \\ y'(t) = y(t)\left(-a_3 + \dfrac{\sigma_3 x_1(t)}{1+d_1x_1(t)+e_1x_2(t)+\mu_1 y(t)} + \dfrac{\sigma_4 x_2(t)}{1+d_1x_1(t)+e_1x_2(t)+\mu_1 y(t)}\right), \end{array}\right\} \\ t \neq (n+\tau-1)T, \qquad t \neq nT, \\ \left.\begin{array}{l} x_1(t^+) = (1-p_1)x_1(t), \\ x_2(t^+) = (1-p_2)x_2(t), \\ y(t^+) = (1-p_3)y(t), \end{array}\right\} \quad t = (n+\tau-1)T, \\ \left.\begin{array}{l} x_1(t^+) = x_1(t), \\ x_2(t^+) = x_2(t), \\ y(t^+) = y(t)+q, \end{array}\right\} \quad t = nT, \\ (x_1(0^+), x_2(0^+), y(0^+)) = (x_{01}, x_{02}, y_0). \end{cases} \tag{2}$$

Naji and Balasim (2007) studied the system (2) when $p_1 = p_2 = p_3 = q = 0$. They analyzed the dynamical behaviors of this system. If we take $\gamma_1 = \gamma_2 = \mu_1 = \mu_2 = 0$, then the system (1) can be reduced to the following Holling-type II two-prey and one-predator system with impulsive perturbations.

$$\begin{cases} \left.\begin{array}{l} x_1'(t) = x_1(t)\left(a_1 - b_1x_1(t) - c_1x_2(t) - \dfrac{\sigma_1 y(t)}{1+d_1x_1(t)+e_1x_2(t)}\right), \\ x_2'(t) = x_2(t)\left(a_2 - b_2x_2(t) - c_2x_1(t) - \dfrac{\sigma_2 y(t)}{1+d_2x_1(t)+e_2x_2(t)}\right), \\ y'(t) = y(t)\left(-a_3 + \dfrac{\sigma_3 x_1(t)}{1+d_1x_1(t)+e_1x_2(t)} + \dfrac{\sigma_4 x_2(t)}{1+d_2x_1(t)+e_2x_2(t)}\right), \end{array}\right\} \\ t \neq (n+\tau-1)T, \qquad t \neq nT, \\ \left.\begin{array}{l} x_1(t^+) = (1-p_1)x_1(t), \\ x_2(t^+) = (1-p_2)x_2(t), \\ y(t^+) = (1-p_3)y(t), \end{array}\right\} \quad t = (n+\tau-1)T, \\ \left.\begin{array}{l} x_1(t^+) = x_1(t), \\ x_2(t^+) = x_2(t), \\ y(t^+) = y(t)+q, \end{array}\right\} \quad t = nT, \\ (x_1(0^+), x_2(0^+), y(0^+)) = (x_{01}, x_{02}, y_0). \end{cases} \tag{3}$$