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Application of response surface methodology and central composite rotatable design for modeling the influence of some operating variables of a Multi-Gravity Separator for coal cleaning

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Abstract

In this study, the application of response surface methodology (RSM) and central composite rotatable design (CCRD) for modeling the influence of some operating variables on the performance of a Multi-Gravity Separator (MGS) for coal cleaning was discussed. Four operating variables of MGS, namely drum speed, tilt angle, wash water and feed solids were changed during the tests based on the CCRD.

In order to produce clean coal with MGS, mathematical model equations were derived by computer simulation programming applying least squares method using MATLAB 7.1. These equations that are second-order response functions representing ash content and combustible recovery of clean coal were expressed as functions of four operating parameters of MGS. Predicted values were found to be in good agreement with experimental values (R^2 values of 0.84 and 0.93 for ash content and combustible recovery of clean coal, respectively).

This study has shown that the CCRD and RSM could efficiently be applied for the modeling of MGS for coal and it is economical way of obtaining the maximum amount of information in a short period of time and with the fewest number of experiments.

Keywords: Central composite rotatable design; Response surface methodology; MGS

1. Introduction

The Multi-Gravity Separator (MGS) represents the latest development in fine grain mineral concentration. The parameters that affect the performance of MGS are the drum speed, tilt angle, shakes amplitude, shakes frequency, wash water and feed solids [1]. The success of concentration with MGS depends on the selection of suitable parameter levels and minerals. The optimization of these parameters requires many tests. The total number of experiments required can be reduced depending on the experimental design technique [2].

Process engineers want to determine the levels of the design parameters at which the response reaches its optimum. The optimum could be either a maximum or a minimum of a function of the design parameters. One of the methodologies for obtaining the optimum results is response surface methodology (RSM) [3].

It is essential that an experimental design methodology is very economical for extracting the maximum amount of complex information, a significant experimental time saving factor and moreover, it saves the material used for analyses and personal costs [4].

The objective of this study was to establish the functional relationships between the some operating parameters of MGS, namely drum speed, tilt angle, wash water and feed solid and, ash content and combustible recovery of clean coal for Yenicubuk/Turkey lignite coal. In the

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following sections, the RSM and requirements for CCRD and its applications for modeling the influence of some operating variables on the performance of a MGS for coal from Yenicubuk/Turkey lignite coal are discussed.

2. Response surface methodology (RSM)

RSM is a collection of statistical and mathematical methods that are useful for the modeling and analyzing engineering problems. In this technique, the main objective is to optimize the response surface that is influenced by various process parameters. RSM also quantifies the relationship between the controllable input parameters and the obtained response surfaces [3].

The design procedure for the RSM is as follows [5]:

(i) Designing of a series of experiments for adequate and reliable measurement of the response of interest.
(ii) Developing a mathematical model of the second-order response surface with the best fittings.
(iii) Finding the optimal set of experimental parameters that produce a maximum or minimum value of response.
(iv) Representing the direct and interactive effects of process parameters through two and three-dimensional (3D) plots.

If all variables are assumed to be measurable, the response surface can be expressed as follows:

$$y = f(x_1, x_2, x_3, \ldots, x_k) \tag{1}$$

where y is the answer of the system, and x_i the variables of action called factors.

The goal is to optimize the response variable (y). It is assumed that the independent variables are continuous and controllable by experiments with negligible errors. It is required to find a suitable approximation for the true functional relationship between independent variables and the response surface [5].

3. Central composite rotatable design (CCRD)

The experimental design techniques commonly used for process analysis and modeling are the full factorial, partial factorial and central composite rotatable designs. A full factorial design requires at least three levels per variable to estimate the coefficients of the quadratic terms in the response model. Thus for the four independent variables 81 experiments plus replications would have to be conducted [6]. A partial factorial design requires fewer experiments than the full factorial. However, the former is particularly useful if certain variables are already known to show no interaction [7,8].

An effective alternative to the factorial design is the central composite rotatable design (CCRD), originally developed by Box and Wilson [6] and improved upon by Box and Hunter [9]. The CCRD gives almost as much information as a three-level factorial, requires much fewer tests than the full factorial and has been shown to be sufficient to describe the majority of steady-state process responses [8,10,11].

The number of tests required for the CCRD includes the standard 2^k factorial with its origin at the center, $2k$ points fixed axially at a distance, say β, from the center to generate the quadratic terms, and replicate tests at the center; where k is the number of variables. The axial points are chosen such that they allow rotatability [9], which ensures that the variance of the model prediction is constant at all, points equidistant from the design center. Replicates of the test at the center are very important as they provide an independent estimate of the experimental error. For four variables, the recommended number of tests at the center is six [9]. Hence the total number of tests required for the four independent variables is $2^4 + (2 \times 4) + 6 = 30$ [8,9].

Once the desired ranges of values of the variables are defined, they are coded to lie at ± 1 for the factorial points, 0 for the center points and $\pm\beta$ for the axial points. The codes are calculated as functions of the range of interest of each factor as shown in Table 1.

When the response data are obtained from the test work, a regression analysis is carried out to determine the coefficients of the response model ($b_1, b_2, \ldots, b_n$), their standard errors and significance. In addition to the constant (b_0) and error (ε) terms, the response model incorporates [8]:

- Linear terms in each of the variables ($x_1, x_2, \ldots, x_n$).
- Squared terms in each of the variables ($x_1^2, x_2^2, \ldots, x_n^2$).
- First-order interaction terms for each paired combination ($x_1x_2, x_1x_3, \ldots, x_{n-i}x_n$).

Thus for the four variables under consideration, the response model is

$$y = (b_0 + \varepsilon) + \sum_{i=1}^{4} b_i x_i + \sum_{i=1}^{4} b_{ii} x_i^2 + \sum_{i=1}^{4} \sum_{j=i+1}^{4} b_{ij} x_i x_j \tag{2}$$

The b coefficients, which should be determined in the second-order model, are obtained by the least squares method. In general Eq. (2) can be written in matrix form

$$Y = bX + \varepsilon \tag{3}$$

Table 1
Relationship between coded and actual values of a variable [12]

Code	Actual value of variable
$-\beta$	$x_{\min}$
-1	$[(x_{\max} + x_{\min})/2] - [(x_{\max} - x_{\min})/2\alpha]$
0	$(x_{\max} + x_{\min})/2$
$+1$	$[(x_{\max} + x_{\min})/2] + [(x_{\max} - x_{\min})/2\alpha]$
$+\beta$	$x_{\max}$

$x_{\max}$ and $x_{\min}$ = maximum and minimum values of x, respectively; $\alpha = 2^{k/4}$; k = number of variables.