International Journal of Mineral Processing 153 (2016) 29–43

Contents lists available at ScienceDirect

International Journal of Mineral Processing

journal homepage: www.elsevier.com/locate/ijminpro

Size–energy relationship in comminution, incorporating scaling laws and heat

Sudarshan Martins

Research and Technology Development, Metso Minerals, 795 George-V, Lachine, Quebec H8S 2R9, Canada

ARTICLE INFO

Article history:
Received 10 March 2016
Received in revised form 30 May 2016
Accepted 31 May 2016
Available online 5 June 2016

Keywords:
Size–energy relationship
Fractal geometry
Characteristic size
Efficiency
Work index
Energy

ABSTRACT

The size–energy relationships in comminution have been the subject of much thought. One of the first theories, developed by Rittinger, postulated that the energy dissipated by the process is proportional to the new surface area generated. By incorporating a fractal description of the surface area — a scaling law — to Rittinger's theory, it is demonstrated that a generalized size–energy relationship, consistent with Hukki's generalized form, is the result. All the parameters required by the model are measureable, including the exponents. The model finds that the 80% passing size, commonly used by size–energy models such as Bond, is inappropriate; smaller particle sizes must be used in energy–size models to properly represent the particle distribution, as required by the Mean Value Theorem. The model includes a definition of the work index and efficiency, based on physical parameters, notably the feed and product temperatures. A discussion of the theory and topics of further investigation is presented. Finally, the properties of several minerals are tabulated.

© 2016 Elsevier B.V. All rights reserved.

1. Introduction

The size–energy relationships in comminution — comminution being the mechanical breakdown of solids into smaller particles without changing their state of aggregation — has interested many researchers for over a century. Notable theories were conceived by Rittinger, Kick and Bond (Balaz, 2008; Jankovic et al., 2010; Morrell, 2004; Bond, 1952; Austin, 1973). The basis for Rittinger's work is that the comminution energy dissipation is proportional to the formation of the new surface area. Kick's theory rests on the proportionality of the required energy with the lost volume. As for Bond, the formation of defects drives the consumption of energy. Other theories can be found in the literature. However, commonalities between some theories can be identified (Hukki, 1962). As illustrated in Fig. 1, Hukki states

$$e = W\left(\frac{1}{{r_p}^{f(r_p)}} - \frac{1}{{r_f}^{f(r_f)}}\right) \tag{1}$$

Here, it is proposed that the Rittinger mechanism leads to Hukki's general relationship, as long as the fractal definition of the specific surface energy is adopted. Furthermore, the parameter set required to complete the model is fully measureable, including the Hukki exponent $f(r)$.

Since large thermal energies are inherent to comminution, they are included as part of the analysis — a discussion of the efficiency is not possible if they are excluded.

2. Detailed particle energy

A grinding circuit is a collection of machines that mechanically break ore, so that the valuable mineral can be economically selected out. If a particle, an element of the ore being processed, is followed, its energy can be written as

$$E = E_{KE} + E_{ROT} + E_{PE} + E_T + E_A + E_{EL} + E_{CHEM} \tag{2}$$

where the total energy is the sum of the kinetic energy, both translational and rotational, the gravitational potential energy, the internal thermal energy, the surface energy, the strain energy and the chemical potential energy, respectively. Substituting each terms with its detailed expression leads to

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh + mc_pT + \gamma A + \frac{m}{\rho}\frac{1}{2}E\epsilon^2 + m\mu \tag{3}$$

During the operation of the comminution circuit, each machine will impart some energy to the particle. Specifically, the conventional operation of the machines is meant to increase the strain energy of the particle until a critical value is reached. Then, the strain energy rapidly decays and is partly converted to surface energy — the particle breaks. The increase in elastic energy can involve changes in kinetic, potential,

E-mail address: sudarshan.martins@metso.com.

http://dx.doi.org/10.1016/j.minpro.2016.05.020
0301-7516/© 2016 Elsevier B.V. All rights reserved.

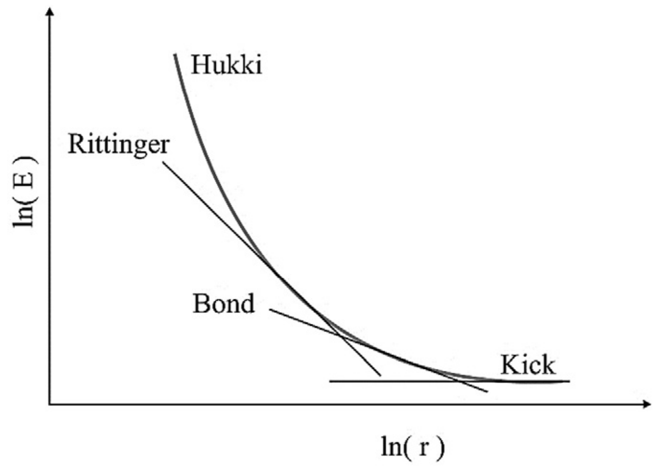

Fig. 1. Comminution size–energy relationships (Thomas and Filippov, 1999).

and thermal energy, such as in a tumbling mill. The heat capacity at constant pressure is used in defining the thermal energy, implying that the total energy described above is in fact an enthalpy, which is appropriate for mass transport problems such as mineral processing (Radziszewski, 2012; Radziszewski and Hewitt, 2015). However, the term total energy will be adopted here, purposefully conflating the two concepts to preserve the focus on size–energy relationships. The total specific energy, the energy per unit mass, is expressed as

$$e = \frac{E}{m} = \frac{1}{2}v^2 + \frac{1}{2}\frac{I}{m}\omega^2 + gh + c_pT + \gamma\frac{A}{m} + \frac{1}{2\rho}E\epsilon^2 + \mu \quad (4)$$

2.1. Energy flow in comminution machines

As the circuit transforms the ore, each machine in the circuit will take a feed, at a specific mass rate or tonnage, and produces a product while consuming some power, as well as generating heat and other losses including sound, vibration and wear. This process is illustrated in Fig. 2.

The specific energy, as described by Eq. (4), describes the particle at all times as it evolves within the circuit, however some terms vanish or can be neglected at certain points in the circuit. At the feed and the discharge of each machine, there is no elastic energy as no stress is applied to the particles at these points — this only happens inside the machines. Moreover, the kinetic and potential energy are well behaved and are generally constant. They will be neglected without any loss of generality. Finally, no bulk material transformations, such as chemical reactions, occur at the feed or the discharge — the chemical potential can be disregarded, leaving the thermal and surface energy terms for the

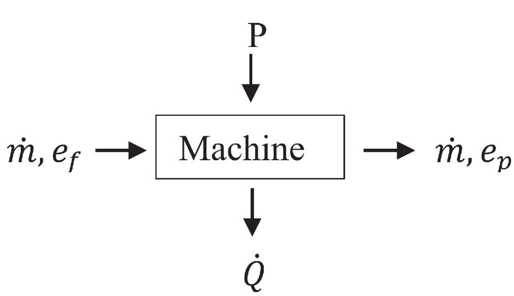

Fig. 2. Comminution machine energy flow.

specific energy,

$$e = c_pT + \gamma\frac{A}{m} \quad (5)$$

The feed and the product consist of more than just a single particle. A large population of particles, with a measurable distribution, flows through the circuit. The size distribution density of the mass fraction is given by $g_{f,p}(x)$, with

$$\int_0^\infty g_{f,p}(x)dx = 1 \quad (6)$$

This is related to the mass fraction distribution $G_{f,p}(x)$ since

$$\frac{dG_{f,p}(x)}{dx} = g_{f,p}(x) \quad (7)$$

The specific energy must account for this distribution, particularly since certain terms are size dependent. Therefore, the specific energy is integrated over the distribution

$$e = \int_0^\infty g_{f,p}(x)\left[c_pT + \gamma\frac{A}{m}\right]dx \quad (8)$$

This is expanded to

$$e = c_pT\int_0^\infty g_{f,p}(x)dx + \int_0^\infty g_{f,p}(x)\left[\gamma\frac{A}{m}\right]dx \quad (9)$$

Using Eq. (6) and expressing the mass as ρV, this is simplified to

$$e = c_pT + \frac{\gamma}{\rho}\int_0^\infty g_{f,p}(x)\frac{A}{V}dx \quad (10)$$

The area and the volume depend on the size of the particle — the remaining part of the integral is evaluated and re-expressed by making use of the Mean Value Theorem for Integrals, as outlined in the Appendix A. In doing so, the specific energy reduces to

$$e = c_pT + \frac{\gamma}{\rho}\frac{A}{V}(r) \quad (11)$$

where r is the characteristic size. The characteristic size is defined such that the integral expressed in Eq. (8) is exact and the surface energy density γ is constant. The selection of r is addressed in Section 3.3.

2.2. Fractal area

Comminution processes do not break particles cleanly — a rough and irregular surface area is produced, having properties similar to fractals. Because of this, fractal geometry is used to describe surfaces (Cox and Wang, 1993; Borodich, 1999; Perfect, 1997). In keeping with fractal geometry, the area scales with the size of the particle (Gongbo and Xiaohe, 1993)

$$A = k_Ar^d \quad 2 \le d < 3 \quad (12)$$

In regular geometry, d is strictly 2. In fractal geometry, the dimension relating surfaces to the length scale r can vary continuously between 2 and 3. The volume, unlike the area, scales as

$$V = k_Vr^3 \quad (13)$$

since fractal behavior has not been observed for solid volumes such as

Download English Version:

https://daneshyari.com/en/article/213690

Download Persian Version:

https://daneshyari.com/article/213690

Daneshyari.com