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Motion of partially contaminated bubbles in power-law liquids: Effect of wall retardation

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ABSTRACT

Momentum transfer characteristics of partially contaminated confined bubbles in power-law non-Newtonian fluids are numerically investigated within the framework of the spherical stagnant cap model. The solver is thoroughly benchmarked by comparing the present results with existing literature counterparts. Further extensive new results are obtained in the range of conditions as: Reynolds number, *Re*: 0.1–200; power-law behavior index, $n = 0.2$–1.6; stagnant cap angle, α: 0°–180° and confinement ratio, λ: 0.2–0.5. Briefly results indicate that the size of recirculation wake at the rear end of bubble decreases with the increasing confinement ratio and/or with the decreasing Reynolds number and/or with the decreasing contamination angle and/or with increasing power-law behavior index. The wall retardation suppresses sudden rise in the surface pressure coefficient at leading edge of the stagnant cap of contaminated bubbles in the case of highly shear-thinning fluids. The total drag coefficients of partially contaminated confined bubbles decrease with decreasing confinement ratio and/or with the decreasing power-law behavior index and/or with the decreasing cap angle. Furthermore, unlike in the case of motion of unconfined bubbles in power-law fluids, the crossover Reynolds number do not exist in C_d vs. *Re* curves with respect to the power-law behavior index because of the domination of wall retardation effects.

1. Introduction

Bubbles are ubiquitous in many chemical, biochemical and mineral processing industries. Some applications include mineral separation/purification by flotation, production of polymeric alloys and ceramics, pulp and paper suspensions, bubble columns, paints and detergents, wastewater treatment, pharmaceutical productions, food processing and preservation. Further the presence of contaminants (or impurities or surfactants) is unavoidable in majority of aforementioned applications. These contaminants significantly decrease the mobility and hence motion of bubble and due to this reduced mobility, the experimental results on the drag behavior of bubbles/drops do not obey Hadamard (1911) – Rybzynski (1911) theory. In order to account the effect of contaminants on motion of bubbles/drops, the spherical stagnant cap model is extensively used in the literature. According to this model, insoluble contaminants can be adsorbed on the surface of the bubble and because of the surface advection caused by the main flow, these so-adsorbed contaminants move from the front stagnation point to the rear end of the bubble. The contaminants accumulated at the rear end form a spherical stagnant cap which is immobile and do not allow the stresses to transmit to the interior of the bubble; however the rest of the bubble surface remains mobile. The angle of the stagnant cap is measured from the rear stagnation point and is referred to as stagnant cap angle (α). By further increasing the concentration of contaminants, gradually the entire bubble surface will be covered by contaminants and the bubble behaves as a solid particle. Steps involved in the formation of spherical stagnant cap are schematically shown in Fig. 1. The kinetics of stagnant cap formation depends on the adsorption/desorption and convection rates of contaminants over the bubble interface (Krzan et al., 2004, 2007). This phenomenon is well supported by many experimental/numerical studies in the literature (Levich, 1962; Clift et al., 1978; Sadhal et al., 1996; McLaughlin, 1996; Cuenot et al., 1997; Wang et al., 1999, 2002; Ponoth and McLaughlin, 2000; Liao and McLaughlin, 2000; Liao et al., 2004; Palaparthi et al., 2006).

On the other hand, many polymeric solutions obey non-Newtonian rheological characteristics including power-law shear thinning and shear-thickening behavior (Chhabra and Richardson, 2008). In general, not only the contamination (surfactants) affect the motion of the bubbles but also the presence of surrounding bubbles (swarms of bubbles), wall retardation due to the presence of confining container wall and rheology of the surrounding continuous liquid will also affect the overall momentum transfer characteristics of the bubble motion. Thus in our previous works, effects of the surrounding bubbles on the overall momentum transfer characteristics of partially contaminated bubble swarms (Nalajala et al., 2014) and effects of power-law type non-Newtonian rheology of surrounding continuous fluids on the flow and drag behavior of partially contaminated unconfined bubbles (Kishore

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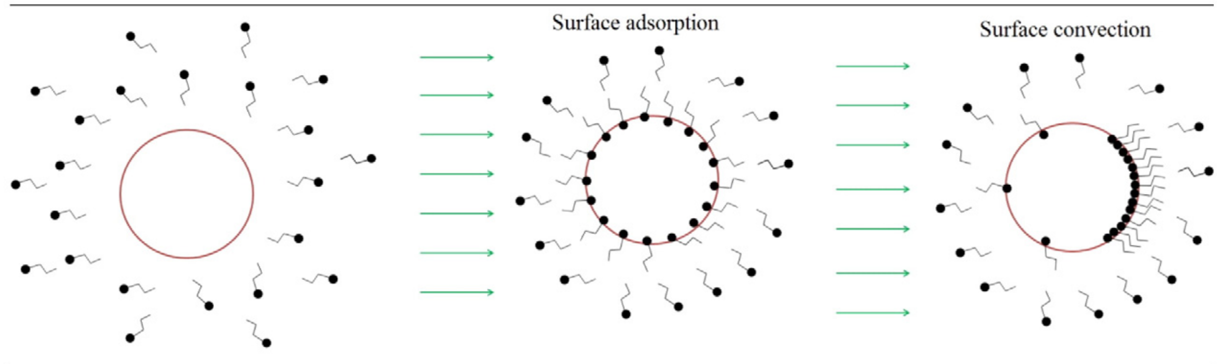

Fig. 1. Schematic representation of formation of spherical stagnant cap.

et al., 2013; Nalajala and Kishore, 2014a, 2014b) are presented. Therefore, in this work, additional effects of confining wall retardation on the overall momentum transfer characteristics of partially contaminated confined bubbles in surfactant-laden power-law type non-Newtonian fluids are delineated.

2. Previous work

The drag force experienced by partially contaminated unconfined bubbles/drops within the framework of stagnant spherical cap model is perhaps first generalized by Sadhal and Johnson (1983) in the creeping flow limit. They proposed an analytical expression for the drag force of contaminated bubbles and drops without considering the non-linear effect. Their correlation reduces to the Levich (1962) solution for the case of unconfined clean spherical bubbles and to the Stokes drag for fully contaminated bubbles (i.e., rigid sphere) in an infinite expanse of quiescent Newtonian liquid. He et al. (1991) revisited this problem by taking account of non-linear effects. Through experiments and numerical simulations Fdhila and Duineveld (1996), validated the stagnant cap model in the limit of moderate Reynolds numbers ($Re = 50$–200). The effect of surfactants on deforming bubbles in quiescent water is numerically investigated by McLaughlin (1996) for Reynolds number up to $Re = 600$. Later many researchers (Cuenot et al., 1997; Ponoth and McLaughlin, 2000; Liao and McLaughlin, 2000; Liao et al., 2004; Palaparthi et al., 2006; Takemura and Yabe, 1999) carried out extensive experimental and/or numerical studies to support the accuracy and reliability of stagnant cap model over wide range of conditions. Takemura and Matsumoto (2000) conducted experiments on dissolution rate of spherical carbon dioxide bubbles in strong alkaline solutions of sodium hydroxide with simultaneous chemical reactions. They also reported numerical results by solving complete Navier–Stokes equations and convection–diffusion equations for bubble dissolving in alkaline solution with simultaneous non-equilibrium reaction at the bubble surface within the framework of the stagnant cap model. The agreement between their experimental and numerical drag coefficients and Sherwood numbers is found to be satisfactory. Zholkovskij et al. (2000) investigated hydrodynamics of stagnant cap formation at low Reynolds numbers and found that the bubble velocity is strongly dependent on the time because of the non-stationary process of stagnant zone formation. Zhang and Finch (2001) experimentally investigated unsteady rising of contaminated bubbles and observed that the transient distance to reach steady state is of the order of few meters at low concentrations of contaminants. Alves et al. (2005) experimentally investigated the effect of bubble contamination on rise velocity and associated mass transfer by using modified stagnant cap model. Sam et al. (1996) in their experimental investigation found that terminal velocity is a function of frother type but not concentration. Kracht and Finch (2010) studied the effect of frother on initial bubble shape and velocity and concluded that the increasing concentration of frother bubble becomes spherical and depends on frother type as well. Thus, on the basis of both experimental and numerical research, adequate information is now available on the momentum transfer characteristics of partially contaminated single bubbles in unconfined surfactant-laden Newtonian liquids at least in the steady axisymmetric flow regime.

On the other hand, available literature on transport phenomena of unconfined contaminated bubbles in quiescent non-Newtonian fluids is limited. Rodrigue et al. (1996, 1997) experimentally and theoretically investigated the rising velocity of contaminated bubbles in Carreau model non-Newtonian fluids in the limit of small Carreau numbers. By the use of a thermodynamic approach and a physical approximation, Rodrigue et al. (1999) further studied the rise of bubbles in power-law and Carreau model type non-Newtonian fluids in the limit of small Reynolds number. The effect of concentration of contaminants on terminal velocity, shape and drag coefficients of bubbles in contaminated power-law liquids is experimentally investigated by Tzounakos et al. (2004). According to them, the rise velocity and surface mobility of bubble are strongly affected by concentration of contaminants; however bubble shape is found to be independent of concentration of contaminants. Recently Kishore et al. (2013), and Nalajala and Kishore (2014a, 2014b) numerically investigated effects of contaminants and power-law type non-Newtonian fluid rheology on momentum transfer characteristics of unconfined partially contaminated bubbles over wide range of pertinent conditions. Finally, to the best of authors' knowledge, no studies are available dealing with the numerical investigation of flow and drag phenomena of partially contaminated bubbles confined in tubes filled with power-law liquids even in the low Reynolds number region, let alone moderate to large values of Reynolds number. Therefore, this work is aimed to fill this gap in the literature over the range of conditions such as Reynolds number, Re: 0.1–200; power-law index, n: 0.2–1.6; confinement ratio, λ: 0.2–0.5 and stagnant cap angle, $\alpha = 0°$–180°.

3. Problem statement and mathematical formulation

Consider relative motion between a bubble (of diameter, d) and a contaminated viscous fluid in a confined container. The relative motion is schematically represented in Fig. 2. The bubble is located along the central axis of a long cylindrical tube of length L, and diameter D_t. With reference to the location of the bubble, the upstream distance is L_u and the downstream distance is L_d. Further an incompressible surfactant-laden power-law fluid is flowing in the tube with an inlet velocity V_o and pressure P_o. During the course of surfactant-laden power-law fluid flow around the bubble, the contaminants (or surfactants) adsorbed along the surface of the bubble and due to the main convection flow these contaminants further dragged towards the rear of the bubble to form a stagnant cap. The angle of the stagnant cap is measured from the rear stagnation point and is referred to as the spherical stagnant cap angle (α). In order to account for wall retardation effects, the diameter of the tube is varied according to the value of the confinement ratio (λ) which is defined as the ratio between the diameters of the