International Journal of Mineral Processing 145 (2015) 7–16

Contents lists available at ScienceDirect

International Journal of Mineral Processing

journal homepage: www.elsevier.com/locate/ijminpro

Identification of a non-linear dynamic model of the bubble size distribution in a pilot flotation column

A. Riquelme [a], A. Desbiens [a,*], R. del Villar [b], M. Maldonado [c]

[a] Department of Electrical and Computer Engineering, LOOP (Laboratoire d'Observation et d'Optimisation des Procédés), Université Laval, Pavillon Pouliot, Québec City G1V 0A6, Canada
[b] Department of Mining, Metallurgical and Materials Engineering, LOOP (Laboratoire d'Observation et d'Optimisation des Procédés), Université Laval, Pavillon Pouliot, Québec City G1V 0A6, Canada
[c] Department of Metallurgical Engineering, University of Santiago, Chile

ARTICLE INFO

Article history:
Received 22 August 2014
Received in revised form 24 September 2015
Accepted 3 November 2015
Available online 6 November 2015

Keywords:
Flotation column
Wiener model
Non-linear model
Bubble size distribution
Image processing
Hough transform

ABSTRACT

Gas dispersion properties play an important role in flotation as they partially determine the metallurgical performance. The objective of this work is to obtain a dynamic model of the bubble size distribution in a two-phase (air and water) pilot flotation column. The steps are: 1) to measure and count the bubbles from digital images taken by a camera, 2) to estimate a log–normal distribution of the bubble sizes, 3) to estimate a Wiener model whose outputs are the mean and standard deviation of the distribution while the inputs are the superficial gas velocity set-point and the superficial shearing water velocity set-point. The size and number of bubbles in each image are evaluated by a bubble detection technique based on circular Hough transform (CHT). This technique allows overcoming issues related to the detection of large single bubbles as well as clusters. Tests are carried out in the laboratory flotation column using different concentrations of frother and air flow rates. Results are compared with visual counting, as well as with a commonly used bubble detection method based on circular particle detection (CPD). The estimated number of bubbles is very similar to what is obtained with a visual count. Compared to CPD algorithm, CHT significantly improves D_{32} estimation (error of 3% instead of 18% with the former) with a comparable processing time. Then, the mean and standard deviation of a log–normal distribution are estimated by maximizing a likelihood function thereby leading to very good fits for the distributions. Finally, a series of model identification tests are conducted by manipulating the superficial gas velocity set-point and the superficial shearing water velocity set-point (i.e. air and shearing water flow rate set-points through the sparger). A Wiener model is then estimated to predict the corresponding mean and standard deviation of the log–normal distribution. Validation tests confirm the quality of the non-linear model.

1. Introduction

Column flotation is a mineral processing method used to separate minerals based on the difference in surface hydrophobicity. The process consists in injecting air bubbles into a cylindrical vessel (typically 12 to 14 m height) where ground ore slurry is introduced at approximately 2 m from the top. Air is dispersed into bubbles at the bottom of the column through a sparger manifold. Prior to its introduction into the flotation column, the slurry is usually conditioned with proper chemicals rendering the surface of valuable minerals hydrophobic, in other words inducing their attraction to air bubbles. Particles exhibiting hydrophobic surfaces thus attach to the rising air bubbles. These bubble-particle aggregates then rises to the top of the column forming a froth phase rich in the valuable minerals. The overflowing froth at the top of the cell (usually the valuable product) is called the concentrate. Hydrophilic particles, which do not attach to the rising bubbles, settle down and exit the column at the bottom port, forming what is called the tailings.

Gas dispersion properties play an important role in flotation as they partially determine the metallurgical performance. One of the most relevant variable is the bubble surface area flux (S_b), which is usually calculated as a function of the superficial gas velocity (J_G) and the Sauter bubble mean diameter (D_{32}) determined from the bubble size distribution (BSD). The Sauter bubble diameter is defined as the diameter of a set of identical size bubbles having the same total surface and volume of the original BSD. After some simplifications the following formula is obtained:

$$D_{32} = \sum_{n}^{i} d_i^3 / \sum_{n}^{i} d_i^2 \qquad (1)$$

where d_i is the i^{th} observed bubble diameter and n is the number of counted bubbles. S_b actually represents the rate of bubble surface available for particle collection. On this regard, it is worth noting that Yianatos and Contreras (2010) developed a model for estimating the

* Corresponding author.
E-mail address: desbiens@gel.ulaval.ca (A. Desbiens).

http://dx.doi.org/10.1016/j.minpro.2015.11.003
0301-7516/

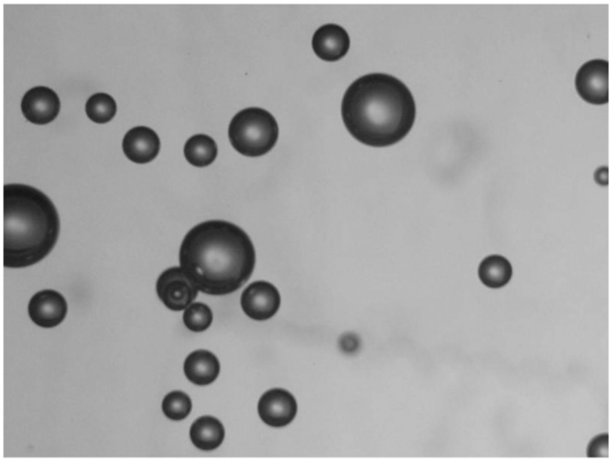
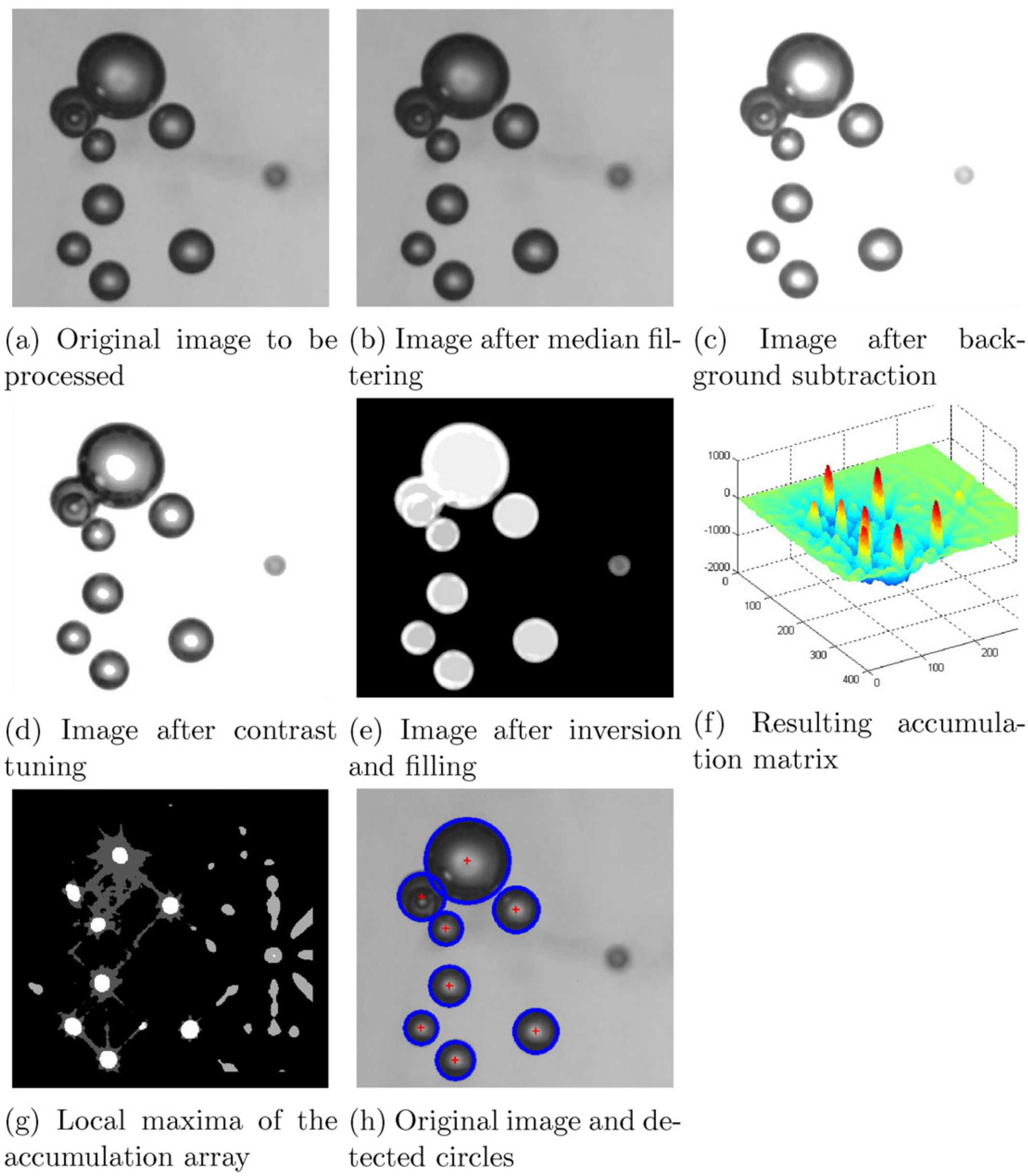


Fig. 1. Sample image of bubbles.

carrying capacity (maximum bubble coverage by particles) of a given flotation machine as a function of the bubble Sauter mean diameter and S_b, among some other variables.

However, a given D_{32} can be obtained with dissimilar shapes of bubble size distributions (Maldonado et al., 2008), thus leading to multiple possible flotation recoveries. This is in accordance with the work by Heiskanen (2000) suggesting that matching the bubble size distribution to the particle size distribution of the valuable mineral would increase its recovery. The same concept also applies to the S_b, as a same value can be obtained from different J_G and D_{32} combinations. To control the entire BSD thus becomes essential to investigate the relationship between the flotation kinetics and hydrodynamic characteristics of the process. In order to design such a control strategy a proper measurement for BSD is required. Once this measurement is obtained, it would then be possible to estimate a dynamic model related to the manipulated variables available for control purposes.

It is thus clear that a reliable technique for measuring bubble size diameters is necessary. Rodrigues and Rubio (2003) reviewed different methods presently used to estimate the BSD, which include drift flux



(a) Original image to be processed (b) Image after median filtering (c) Image after background subtraction

(d) Image after contrast tuning (e) Image after inversion and filling (f) Resulting accumulation matrix

(g) Local maxima of the accumulation array (h) Original image and detected circles

Fig. 2. Original image and results after each step of bubble detection processing.