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Self-similarity of upgrading parameters used for evaluation of separation results

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ABSTRACT

A proper characterization of upgrading results requires two material balance parameters, usually plotted as upgrading curves. It was presented in the paper that such pairs of separation result parameters have a certain degree of self-similarity defined as an extent to which one parameters is a part of the other. Some pairs of upgrading parameters have a significant degree of self-similarity. Upgrading curves based on such parameters are unsuitable for a simple statistical evaluation of separation results, especially when the determination coefficient R^2 is used for this purpose. This is so because the R^2 value can be as high as almost 1, even for highly scattered original data points. It results from comparison of nearly two identical parameters. It was shown in the paper that, for example, the upgrading curve relating the content of the considered component in concentrate less the content of the considered component in tailing and the content of the considered component in concentrate have the self-similarity approaching 99% on a 0 to 100% scale proposed in the paper. Other upgrading curves used in mineral processing have medium (between 50 and 60%) self-similarity, while the Fuerstenau plot, relating recovery of the useful component in concentrate and recovery of the remaining components in the tailing, has a relatively low self-similarity which changes with recovery from 0 to about 50%.

1. Introduction

During mineral processing operations the starting material, called the feed, consisting of two or more components, is split into two or more products. Even though a separation system is complex, it can always be considered in a practical way, that is as a feed consisting of one component and the rest while due to separation, two products, concentrate and tailing are created. For simplicity such a system will be considered in this work and the feed will be represented by symbol f, concentrate by c and tailing by t. The components of the system will be denoted as component A and the rest material as R or r. The system with the feed, concentrate and tailing has four material variables: mass of component A in the feed $m_{A,f}$, mass of component R in the feed $m_{R,f}$, mass of component A in concentrate $m_{A,c}$ (mass of component A in tailing $m_{A,t}$ can be calculated since $m_{A,f}=m_{A,c}+m_{A,t}$), and mass of component R in tailing $m_{R,t}$ (mass of component R in concentrate $m_{R,c}$ can be calculated since $m_{R,f}=m_{R,c}+m_{R,t}$). The considered system is depicted in Fig. 1.

A proper plotting of the separation results for the considered system, especially for statistical considerations, should be based on original variables, for instance $m_{A,c}$ versus $m_{R,t}$ (other options are $m_{A,c}$ versus $m_{R,c}$, $m_{A,t}$ versus $m_{R,t}$, and $m_{R,c}$ versus $m_{A,t}$). However, in mineral processing, for practical purpose other parameters, expressed in per cent, are used, including yield (for concentrate: $\gamma_c=100\%\cdot(m_{A,c}+m_{R,c})/(m_{A,f}+m_{R,f})$, for tailing $\gamma_t=100\%\cdot(m_{A,t}+m_{R,t})/(m_{A,f}+m_{R,f})$), recovery (for instance for component A in concentrate: $\varepsilon_{A,c}=100\%\cdot(m_{A,c}/m_{A,f})$), and content (assay, grade) (for instance component A in concentrate: $\beta_{A,c}=100\%\cdot m_{A,c}/(m_{A,c}+m_{R,c})$). Contents are especially important because they are used for the so-called technological characterization and evaluation of separation results (Fig. 2).

It is obvious that yield γ, recovery ε and content β, are not original variables of a separation system but a combination of original masses of different components in different products and the feed. As a result, each variable from the list of different γs, βs, and εs contains a certain amount of another variable. Thus, the considered variables γ, β, ε and other variables obtained by their combination (for instance upgrading index $K=\beta/\alpha$) are always, to a certain extent, self-similar. Therefore, plotting two selected variables from the inventory of different γs, βs, and εs leads to certain self-similarity of the Cartesian axes on which the self-similar variables are plotted. The goal of this paper is to evaluate the extent of self-similarity of selected upgrading parameters which are used for plotting upgrading curves and for statistical analysis of separation results, especially those encountered in mineral processing. In this work self-similarity for two upgrading result parameters is defined as an extent to which one parameter is a part of the other parameter. This definition is analogical to self-similarity term used in mathematics to describe fractal property of an object which can be exactly or approximately similar to a part of itself. In the case of fractals, the whole has the same shape as one or more of the parts.

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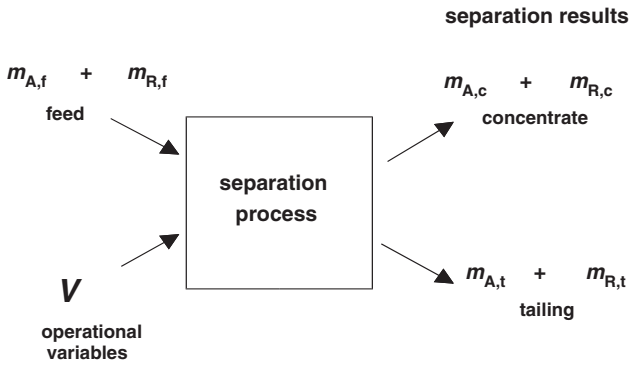

Fig. 1. In separation systems the initial material (feed, f) consisting of a useful component A and the rest R is split into products (concentrate c, and tailing, t) differing in composition. The feed and products are characterized by mass m of components. Content of feed and operational variables influence the process.

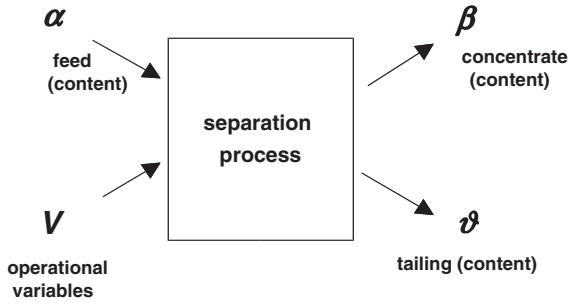

Fig. 2. In mineral processing separation results are usually considered in technological terms of content of the considered component in concentrate β, tailing ϑ, and the feed α. Numerous upgrading parameters can be expressed by different combinations of α, β and ϑ.

2. Self-similarity of selected upgrading parameters

To assess the degree of self-similarity of parameters used for plotting upgrading curves, the parameter plotted on the y axis is split into a part identical with the parameter occupying the x axis and the rest, which represents the non-similar part:

$$y = f(x),\ y = S + N, x = S \tag{1}$$

where the similar part is denoted as S and the non-similar part as N.

Evaluation of self-similarity of upgrading parameters and splitting upgrading parameters into the S and N parts is not a simple task. One of the possible approaches is to use a set of separation results which can be easily approximated with a one-fitting parameter formula. For the purpose of this work we used hypothetical separation results presented in Table 1, which were approximated with the same one-fitting parameter "a" using the so-called Fuerstenau upgrading curve (Drzymala and Ahmed, 2005). The mathematical formulas relating the upgrading curve axis parameters and fitting parameter (a) are different for particular upgrading curves because each plot is based on a different and unique pair of upgrading parameters (Duchnowska and Drzymala, 2011). Table 2 shows selected upgrading parameters arranged into pairs forming different upgrading curves. Table 2 also provides mathematical formulas of the upgrading parameters expressed in terms of principal parameters α, β, ϑ and equations for one-fitting parameter a.

In this work the self-similarity will be assessed by using a self-similarity index (S_S) defined as

$$S_S = 100\frac{\mathrm{abs}(S)}{\mathrm{abs}(S) + \mathrm{abs}(N)} \tag{2}$$

where S and N are the already mentioned similar and non-similar parts of a given upgrading parameter and abs means absolute value.

2.1. The Stepinski upgrading curve

This upgrading curve relates two principal upgrading parameters considered in the so-called technological evaluation of separation results, which are the content of the considered component in the concentrate β and in the tailing ϑ (α is constant) (Stepinski, 1955, 1958, 1965; Drzymala, 2006–l2008, 2007). The Stepinski plot and its properties are shown in Fig. 3. The data points from Table 1 were approximated using equation with a given in Table 2 for the Stepinski I upgrading curve and in Eq. (3) using $a = 104$. The value of the fitting parameter a was determined with a computer software capable of handling nonlinear correlations.

The equation relating both parameters of the Stepinski I upgrading curve, that is β and ϑ, with one-fitting parameter a is:

$$\vartheta = \frac{\beta(100-a)}{\beta - a}. \tag{3}$$

Eq. (3) can be rearranged into N and S parts:

$$\vartheta = \beta + \frac{\vartheta(\vartheta-100)}{\vartheta-100+a} \tag{4}$$

in which the S part is

$$S = \beta \tag{5}$$

and part N is

$$N = \frac{\vartheta(\vartheta-100)}{\vartheta-100+a}. \tag{6}$$

The relations between N and β as well as S and β were plotted in Fig. 4.

Having the values of N and S, the self-similarity index can be calculated. For the Stepinski curve it is shown in Fig. 5. The self self-similarity index is about 50% indicating a high self-similarity of parameters β and ϑ.

Table 1
Mass balance of a hypothetical set of separation results used in calculation of self-similarity of upgrading parameters forming upgrading curves.

Product	$\gamma_c = \gamma\%$	$\beta_{A,c} = \beta\%$	$\vartheta_{A,t} = \vartheta\%$	$\varepsilon_{A,c} = \varepsilon\%$	$\varepsilon_{R,t} = \varepsilon_r\%$	Parameter β-ϑ
Product 1	13.09	23.12	1.12	74.59	89.52	22.00
Product 1 + 2	18.64	18.16	0.75	80.63	84.11	17.41
Product 1…3	24.29	14.49	0.63	88.74	78.36	13.85
Product 1…4	49.84	7.72	0.30	95.36	52.09	7.42
Product 1…5	60.06	6.51	0.23	97.50	41.51	6.28
Feed (product 1…5 + tailing)	100.00	4.00 (α)	0.00	100.00	00.0	4.00