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Impact of frother on the terminal velocity of small bubbles

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ABSTRACT

Using a circulating flow that balances buoyancy and drag, small bubbles (<1 mm) are held in a column that classifies them according to their terminal velocities. Without frother, the terminal velocities fall between the values predicted by Hadamard–Rybczynski for fluid spheres, and those predicted by Stokes for hard spheres. Although it is commonly believed that industrial surfactants have little to no impact on such small bubbles, this study demonstrates a trend comparable to that of larger bubbles, namely that the addition of frother can retard the bubbles even beyond the predictions of the hard sphere model. Hence the motion of small bubbles appears to be impeded by mechanisms similar to those acting on larger bubbles. The frothers studied were MIBC and Dowfroth 250.

1. Introduction

Understanding bubble swarm dynamics is the key to effective control of a variety of processes, mineral flotation being one important example. An expanding knowledge of how bubbles interact one-on-one, as well as globally within a swarm, is helping researchers and operators discover optimal surfactant chemistries and operating strategies. In terms of bubble properties, the most important surfactant in mineral flotation is the frother, added to control (reduce) bubble size and promote froth formation. There is a continuing debate on the general impact that frothers have on elementary bubble properties (Harris, 1982; Pugh, 1996; Grau and Laskowski, 2006; Acuña et al., 2007). A clear understanding of beneficial bubble characteristics is complementary to the design of frothers, as well as the search for optimal operating conditions (Cappuccitti and Finch, 2007). The work here isolates the effect of frothers on bubble terminal velocity of small (<1 mm) bubbles.

2. Theory

2.1. Fluid mechanics

Classical fluid mechanics features two problems relevant to spherical bubble motion through water: (i) the motion of a fluid sphere balanced by buoyant and drag forces, and (ii) the motion of a rigid sphere balanced by these same forces. The former seems to describe the rise of air bubbles through water. However, the spherical regime dominates for small bubbles ($d<1$ mm in water), in which even a small amount of contamination induces a viscous layer at the air–water interface, causing the surface to become 'rigid' (Clift et al., 2005a). The nature of the contamination layer will be discussed in Section 2.2, but for now consider the cases depicted in Fig. 1.

Given the presence of a viscoelastic layer (Fig. 1b), the behaviour of a rising bubble should be between that of an idealised fluid sphere (Fig. 1a), and that of an idealised hard sphere (Fig. 1c), as the layer obstructs the transmission of shear forces from the water to the air. Indeed, terminal velocities can fill the entire spectrum from an ideal fluid sphere all the way to an ideal rigid sphere (and beyond, if other mechanisms are considered, as discussed in Section 2.2).

The terminal velocity of an ideal (Newtonian) fluid sphere rising through another ideal fluid was obtained by Rybczynski (1911) and independently by Handamard (1911) for low Reynolds numbers ($Re<1$) (Dukhin et al., 1998). Taking the viscosity and density of water to be much larger than those of air, the terminal velocity in water becomes:

$$U_{\mathrm{HR}} = \frac{gd^2\rho}{12\eta} \tag{1}$$

where g is the gravitational constant, d is the bubble diameter, ρ is the density of water and η is the viscosity of water. The terminal velocity of a rigid sphere (Fig. 1c) is the classical result of Stokes (1851),

$$U_{\mathrm{St}} = \frac{gd^2\rho}{18\eta} \tag{2}$$

To derive these two equations, $Re<1$ is assumed to simplify (linearize) the Navier–Stokes equations. Bubbles ranging from 0.4 to 1.0 mm in diameter fall in an intermediate regime ($1<Re<100$), but Eqs. (1) and (2) are still essential points of comparison because of

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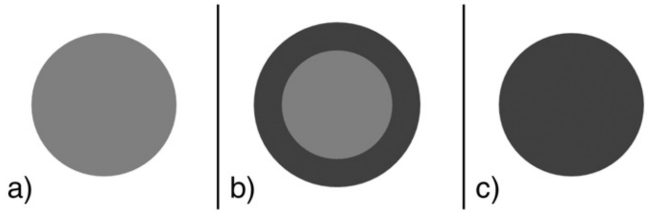

Fig. 1. An idealised spherical bubble, without any contamination at the air–water interface (a). A bubble with a viscous layer separating the air from the water (b). An idealised rigid sphere (c).

their simplicity, and because an analytical solution is inaccessible in this intermediate regime (Dukhin et al., 1998). There are non-linear contributions in the intermediate region and departure from the perfect spherical shape; these affect the terminal velocity.

2.2. The contamination layer

Almost universally added to mineral flotation pulps, frother molecules help form the viscoelastic layer by organizing the adjoining water molecules which become thermodynamically difficult to displace,[1] and the bubble starts to become rigid. There are, however other secondary mechanisms believed to diminish the energy available for upward motion. In fact, terminal velocities may fall below U_{St} (Dukhin et al., 1998); evidently, the obstruction of shear is one means to decrease the terminal velocity, but it cannot be the only means.

The debate over secondary mechanisms has spanned nearly a century (Dukhin et al., 1998). As a bubble increases in speed, it is deformed into an oblate spheroid, storing potential energy according to Hooke's law. When the bubble diameters are below 1 mm, any deformation is linear and results in symmetric oblate spheroids, as confirmed by Duineveld (1995), who observed departure from fore-aft symmetry only above 1.06 mm.

Wobbling is a secondary kinetic phenomenon especially important for large bubbles (d>1 mm). Vertical terminal velocities are further diminished if there is a helical component to the bubble trajectory, constituting another secondary kinetic mechanism. In principle, if a bubble were to spin along its own axis, the spin would take up kinetic energy, further decreasing the terminal velocity. Linear and nonlinear deformation (storage of potential energy), as well as helical motion and wobbling (secondary kinetic mechanisms), are common in bubbles but the organized spinning of surface and/or subsurface layers is not (Clift et al., 2005b).

A spherical bubble experiences shear stress as it rises through the pulp, causing a tension gradient in the longitudinal direction; the frother at the rear of the bubble tends to a compression limit, resulting in a stagnant cap. (The stagnant cap can be considered ultra-viscous.) Therefore the tension gradient is coupled with a frother concentration gradient, in which there is a frother shortage at the front, and a frother surplus at the rear. This results in an ongoing adsorption of frother molecules in the front, with a simultaneous desorption of frother molecules from the rear. This action, due to the coupled tension and concentration gradients, is the Marangoni Effect, and it induces surface perturbations, another secondary phenomenon.

From this discussion, frothers induce two kinds of energy storage: in the viscous layer, shear is prevented from reaching the inner air and is trapped as potential energy; secondly, an additional amount of energy is expressed as surface perturbations and the motion of frother molecules, hence a form of kinetic energy. The potential energy storage is comparable to the action of a mechanical spring or an electronic capacitor, while the kinetic energy storage is comparable to action of a flywheel or an inductor.

[1] There is an entropy gradient pointing radially out of the bubble.

Whenever a material, or more generally a physical space, stores energy in a conservative manner (be it kinetic or potential energy), the continued addition of energy favours dissipation, e.g. friction and the like. A steady terminal velocity is attained only when the rate of energy storage is balanced by the dissipation. *A priori*, the bubble would have been subject to an unsteady force, which is often described in two parts: the basset force and the added mass force (Brennen, 2005). The basset (a.k.a. the 'history') force addresses the viscous effects and the delay in the boundary layer development.

The added mass force is due to the acceleration of the adjacent water (it has the same effect as an increase in the bubble mass). According to Gélinas et al. (2005), frother molecules organize water over some considerable distance (due to H-bonding). Perhaps the added water carried alongside the frother molecules is enough for a marked change in the acceleration as well as the terminal velocity.

3. Experimental setup

To study the effect of frother on the terminal velocities of bubble populations below 1 mm, an experimental apparatus consisting of a trapezoidal column subtended by a surge tank, was placed in cycle with a pump (Fig. 2). The water is pulled from the bottom of the Plexiglas column and sent back up to the surge tank, so that the buoyancy forces pushing the bubbles upward are opposed by the downward drag from the water. When in balance, the net effect is that the bubbles stay at a more-or-less fixed position in the column. Given the trapezoidal shape of the column, different heights have different cross-sectional areas, hence different downward water velocities. Thus bubbles report to a height corresponding to their terminal velocity. Large bubbles have high terminal velocities, so they tend to report near the top (where the cross-sectional area is small); conversely, smaller bubbles have lower terminal velocities, hence they tend toward the lower regions.

Initially 40 L of water (Antofagasta, Chile tap) enters through the input valve, passing through a carbon filter and into the previously empty apparatus. At this point the input valve is closed and the pump is turned on. The water exiting the pump is flow regulated with a valve and orifice plate. Parallel to the orifice plate is a bypass used for cleaning

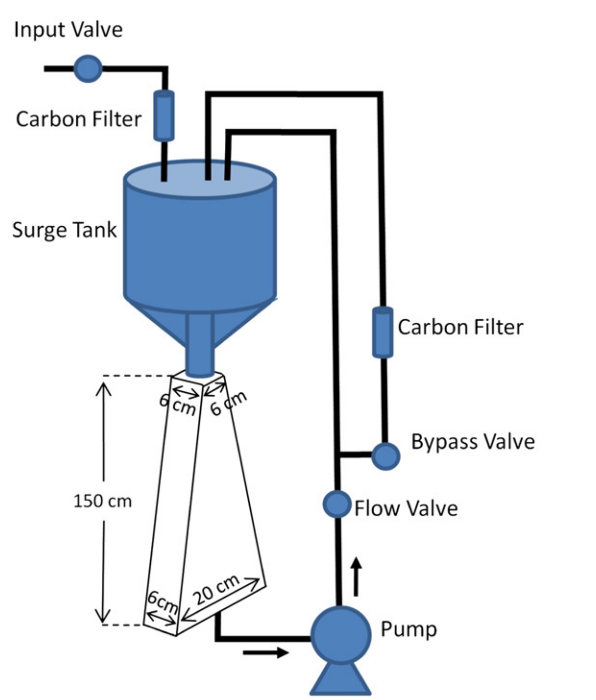

Fig. 2. Schematic of the experimental apparatus. The input and bypass valves are closed during operation, so that water is circulated up directly to the surge tank, and down through the trapezoidal column back to the pump.