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Int. J. Miner. Process. 78 (2006) 110–121
INTERNATIONAL JOURNAL OF MINERAL PROCESSING
www.elsevier.com/locate/ijminpro

Drag on non-spherical particles in power law non-Newtonian media

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Received 8 September 2005; accepted 8 September 2005
Available online 7 October 2005

Abstract

The free settling velocity of cylinders and disks falling in quiescent Newtonian and power law liquids has been measured over wide ranges of experimental conditions of the particle Reynolds number (10^{-5}–~300), power law flow behaviour index (0.31–1) and the length-to-diameter ratio, ~0.4–~14. The corresponding range of sphericity is 0.62 to 0.86. An existing drag expression which has been tested extensively for spherical particles falling in Newtonian and in power law fluids has been slightly modified here for non-spherical particles. In particular, the use of this drag expression necessitates a knowledge of an equal volume sphere diameter (to evaluate the Reynolds number and drag coefficient) and the ratio of the surface area to the projected area of a non-spherical particle. With these modifications, the approach outlined here reproduces the present and the literature data for a wide range of non-spherical particles including cones, prisms, needles, cylinders settling in both Newtonian and power law fluids with reasonable levels of accuracy.

Keywords: non-spherical particles; drag; power law fluids; Sphericity; Reynolds number

1. Introduction

There are numerous situations in process engineering applications when a reliable prediction of the terminal falling velocity of particles in stationary fluids, and/or the fluid dynamic force exerted on a particle placed in moving liquid streams is required. Typical examples include process design calculations for the hydraulic transport of coarse grains using viscous carriers in slurry pipelines, for continuous asceptic processing of large food particles suspended in water-like or viscous Newtonian and non-Newtonian liquids, fixed and fluidized bed reactors, liquid–solid separation and classification equipment and in petroleum engineering applications, etc. (Clift et al., 1978; Coulson and Richardson, 1990; Chhabra, 1993; Chien, 1994; Shah, 1996; Lareo et al., 1997; Wilson et al., 2003; Tang et al., 2004). While it is readily acknowledged that most practical applications entail hindered settling conditions, experience has shown that the knowledge of the single particle falling velocity often serves as a useful starting point. Therefore, over the years, considerable work has been reported on formulating reliable and accurate predictive schemes for estimating the drag force on a particle and/or the free falling velocity of particles in fluids. In spite of the fact that very few applications entail the use of perfectly spherical particles, the bulk of the available literature on this topic relates to spherical particles in both Newtonian and non-Newtonian liquids (Chhabra, 1993; Ceylan et al., 1999; Wilson et al., 2003; Renaud et al., 2004; Kelessidis, 2004, for instance). This is in part

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doi:10.1016/j.minpro.2005.09.003

due to the obvious advantage of the perfect symmetry exhibited by a sphere. Therefore, the literature is inundated with numerical solutions and empirical expressions enabling the prediction of the settling velocity of spheres in Newtonian (Clift et al., 1978; Haider and Levenspiel, 1989; Khan and Richardson, 1987; Hartman et al., 1994) and non-Newtonian liquids (Chhabra, 1990, 1993, 2002; Darby, 2002; Goel et al., 2002; Wilson et al., 2003; Renaud et al., 2004). Suffice to say here that based on a combination of numerical and experimental results, it is now possible to estimate the free falling velocity of a sphere in Newtonian fluids under most conditions of practical interest. On the other hand, the available methods for power law liquids are limited to the values of the sphere Reynolds number of about 1500 and in broad terms, the predictions are somewhat less accurate for power law liquids than that for Newtonian fluids.

In spite of such a high degree of idealization, the results for spherical particles often serve as a useful starting point for developing analogous predictive equations for non-spherical particles. Indeed, scores of empirical expressions of varying forms and complexity are now available in the literature (Chhabra et al., 1999; Yow et al., 2005) which allow the estimation of the free settling velocity of non-spherical but regular shaped particles (cylinders, cones, prisms, bars, discs) in Newtonian media with modest levels of accuracy and reliability (~typically of the order of 25%–30% with maximum errors often being even of the order of 100%–200%). On the other hand, virtually no attempt seems to have been made to formulate analogous expressions for non-spherical particles falling in power law type inelastic non-Newtonian systems, typical of mineral slurries, drilling fluids and food suspensions (Shah, 1996; Lareo et al., 1997; Chhabra and Richardson, 1999). This work is aimed at filling this gap in the currently available body of knowledge.

Extensive experimental results are reported on the free settling velocity of circular cylinders falling in both orientations in quiescent power law liquids encompassing wide ranges of kinematic conditions, particle shape and non-Newtonian liquid characteristics. However at the outset, a terse description of the pertinent studies available in the literature is regarded to be useful for the subsequent treatment of the present results and of those available in the literature.

2. Previous work

The terminal settling velocity (and/or the drag coefficient) of an object is influenced by a large number of variables including its density, size, shape and orientation in addition to the physical and rheological (power law constants) properties of the liquid medium, wall effects, etc. Indeed, the lack of a clear cut description of shape, size and orientation during the settling of a non-spherical object has been (and continues to be) the main impediment in developing universally applicable correlations for settling even in Newtonian fluids (Chhabra et al., 1999; Yow et al., 2005). This difficulty is further accentuated for power law liquids.

From a theoretical standpoint, there has been very little activity in this area and most such attempts are limited to two-dimensional shapes. Thus, for instance, limited numerical results for the steady flow of power law (both shear-thinning and shear-thickening behaviour) fluids past spheroidal particles are now available up to about particle Reynolds number of 100 based on an equal volume sphere diameter (Tripathi et al., 1994; Tripathi and Chhabra, 1995). They reported a slight increase in drag in shear-thinning liquids and slight reduction in drag in shear-thickening fluids with reference to its Newtonian value at low Reynolds numbers. The effect of the power law index was seen to diminish as the Reynolds number was gradually increased. Similarly, scant results are also available on drag coefficient for infinitely long circular cylinders and square bars in cross-flow configuration at low Reynolds numbers (Tanner, 1993; Whitney and Rodin, 2001; Ferreira and Chhabra, 2004) and at moderate values of the Reynolds number (≤40) (D'Alessio and Pascal, 1996; Gupta et al., 2003; Chhabra et al., 2004; Soares et al., 2005). From a practical standpoint, the utility of such results for infinitely long cylinders and square bars is rather limited, for it is virtually impossible to predict a priori as to what value of the ratio (l/d) will constitute an infinitely long cylinder. Very recently, the sedimentation of circular disks oriented normal to the flow in Newtonian and power law fluids has been studied numerically in the Reynolds number (based on disk diameter) range 1 to 100 (Nitin and Chhabra, 2005a, in press). This study clearly showed that the bulk of the drag was due to the pressure component. Furthermore, the predicted values of drag coefficient were found to be in line with an existing correlation (Clift et al., 1978) which is also consistent with the findings of Chhabra et al. (1996). Unfortunately, no attempt seems to have been made to utilize the aforementioned numerical results to develop a single predictive equation (Chhabra, 1996a).

Similarly, only a few experimental studies dealing with the free falling behaviour of non-spherical objects in quiescent power law fluids are available. Reynolds