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Mathematical modeling of shrinkage deformation in eggplant undergoing simultaneous heat and mass transfer during convection-oven roasting

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ABSTRACT

This study aimed to analyze experimentally and by modeling the shrinkage deformation of Japanese eggplant (*Solanum melongena*) during roasting by simultaneous heat and moisture transport model coupled with a structural mechanics model applicable to a body undergoing volumetric changes, as a consequence of moisture removal. Cylindrical slices (45 mm × 10 mm) were roasted for up to 15 min. Roasting experiments were performed using a convection-oven at 250 °C. Several coefficients and parameters required for calculations were experimentally obtained. An implicit finite element method was applied using a FORTRAN program. In this study, an approximately equal reduction rate in moisture content losses and volume shrinkage was observed ($R^2 \geq 0.99$). The volumetric (S_v) and directional shrinkage were successfully estimated by developed empirical equations (e.g. $R^2 > 0.98$ for S_v). The simulation of temperature and moisture distribution undergoing shrinkage deformation confirmed the importance of internal evaporation in the transport model.

1. Introduction

During thermal treatments such as drying, broiling, and roasting, foods undergo deformations that can be characterized by changes in volume, shape, porosity, density, shrinkage and/or collapse phenomena (Ratti, 1994). These changes are of extreme importance in terms of product quality and characterization of mass and heat transfer phenomena. Senadeera (2008) reported that for air-dried food materials, the most important consequence of these changes is volume reduction, coupled with shape and porosity changes and hardness increase. Such phenomena can also be followed by surface cracking. Excessive shrinkage of processed foods is to be avoided, because such physical changes reduce the quality perceived by the end consumer.

The presence of both liquid water and vapor in the solid matrix, the dependence of food transport properties on the local values of temperature and moisture content, the complex fluid-structure interactions determined by air-flow around the sample, and the variation with time of food shape make modeling the drying process rather problematic (Chen, 2007). However, higher temperatures processes are more complicated than simple drying, as the drying rate is faster and the sample surfaces easily exceed 100 °C; thus, rapid evaporation occurs, accompanied by a change of state from rubbery to glassy, which affects the final deformation and shape.

According to Kowalski and Rajewska (2002), the removal of water from a porous solid material such as eggplant, considered in this study, is responsible for the development of a field of contracting stresses in the matrix. In hygroscopic material, there is a large amount of physically bound water, and the material often shrinks during heating. According to Datta (2007), in these materials, below the level of moisture saturation, the internal vapor pressure is a function of moisture level and temperature and is lower than that of pure water. Above this moisture saturation, the vapor pressure is a function of temperature only, and is independent of the moisture level. Thus, above a certain moisture level, all materials behave non-hygroscopically.

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Nomenclature

A	Matrix defined by Eq. (13) [m^{-1}]
a	Nodal displacement vector [m]
B	Strain nodal displacement matrix [m^{-1}]
C_p	Specific heat [kJ kg^{-1} K^{-1}]
D	Elastic stress-strain matrix by Eq. (8) [m^{-1}]
D_w	Moisture diffusivity [m^2 s^{-1}]
da	Nodal displacement vector [m]
du, dv	Displacements in r and z direction, respectively [m]
E_y	Young modulus [Pa]
H_L	Latent heat of vaporization [J g^{-1}]
h_m	Convective surface mass transfer coefficient [m s^{-1}]
h_t	Convective surface heat transfer coefficient [W m^{-2} K^{-1}]
k	Thermal conductivity [W m^{-1} K^{-1}]
k_v	Evaporation rate constant [kg-solid s^{-1} m^{-3} hPa^{-1}]
n	Outward normal unit vector [–]
P_w	Partial pressure of the saturated water vapor in the liquid phase surface of the food [hPa]
P_{we}	Partial pressure of the saturated water vapor in the gas phase of the food [hPa]
r	Radial distance in cylindrical coordinates [m]
S_r, S_z, S_θ	Free shrinkage coefficients in r, z, and θ directions, respectively [–]
S_v	Volumetric shrinkage coefficient [–]
T	Temperature [K]
T_h	Wall temperature [K]
t	Time [s]
U	Displacement vector [m]
V_e	Internal evaporation rate [kg-water s^{-1} m^{-3}]
v_e	Flow velocity of air [m s^{-1}]
W_w	Moisture content [kg-water kg-solid^{-1}]
W_{we}	Equilibrium moisture content [kg-water kg-solid^{-1}]
z	Thickness distance in cylindrical coordinates [m]
β	Rate of air and water in the porous material [–]
ε	Observed strain [–]
ε_0	Initial strain (free hydrostrain) [–]
ε_c	Viscositic strain [–]
ε_S	Elastic strain [–]
ε_s	Emissivity of silicon [–]
ε_{sw}	Emissivity of wrapped silicon with aluminum foil [–]
$\bar{\varepsilon}_c$	Equivalent viscositic strain [–]
$\bar{\varepsilon}'_c$	Equivalent viscositic strain rate [s^{-1}]
ν	Poisson's ratio [–]
ρ_b	Density of material [kg m^{-3}]
ρ_s	Density of porous solid [kg-solid m^{-3}]
σ	Stress [kN m^{-2}]
$\bar{\sigma}$	Equivalent stress [kN m^{-2}]
σ'	Deviatoric stress [kN m^{-2}]
σ_c	Stefan Boltzmann constant [W m^{-2} K^{-4}]
σ_d	Yield stress [kN m^{-2}]
φ	Summary radiant heat transfer coefficient [–]

A porous medium is a solid having pore space that is filled with a gas or liquid. Generally, these pores are interconnected, allowing faster transport of mass and heat through the pores than through the solid matrix (Datta, 2007). Datta claimed that application of transport through porous media in food materials is sparse, perhaps due to the difficulty in obtaining the many process parameters needed, the complexity of such formulations, and the lack of software tools needed to solve the resulting set of equations. In addition, several assumptions have been made in almost all multiphase porous media studies to avoid the computational complexities of a coupled model, such as thermal equilibrium of the solid, liquid and gas (vapor + air) at any location, isotropic behavior of properties due to the lack of detailed information, constant characteristics dimension, negligibility of water evaporation inside the food, and shrinkage (which is commonly ignored, although most foods experience it during processes involving heat and mass transfer).

Mathematical models that predict the distribution of moisture and temperature in eggplant, undergoing simultaneous heat and moisture transfer besides volume changes, have been reported by a number of researchers (Aversa et al., 2011; Brasiello et al., 2013; Mayor and Sereno, 2004). Another approach is the analysis of deformation by applying a structural mechanics model. Yang et al. (2001) developed a model of simultaneous heat and moisture transfer in a cylindrical sample coupled with the virtual work principle applicable to a cylindrical potato undergoing shrinkage deformation in two dimensions. However, they assumed potato as elastoplastic material within a not large strain region that is not appropriate for vegetables.

The elastoplasticity condition assumed by Yang et al. (2001) was reconsidered in studies by Itaya et al. (1995) and Sakai et al. (2002) in which viscoelasticity characteristics were assumed. Visco-elastic materials are those for which the relationship between stress and strain depends on time. Itaya et al. (1995) developed a three-dimensional heat and moisture transfer and visco-elastic hygro-stress formation in a composite body undergoing drying. They considered the interactive influence of non-homogeneity on heat and moisture transfer as well as on strain-stress formation by a three-dimensional transport processes for the first time. Sakai et al. (2002) developed a drying model that takes into account shrinkage deformation in visco-elastic food accompanying changes in moisture content. They discussed in detail the differences between the assumption of potato as an elastoplastic or visco-elastic material, concluding that the assumed visco-elastic characteristics improved the simulation accuracy of shrinkage deformation compared with experimental results, because the inclusion of the viscous modulus contributed in a better theoretically explanation of the non-uniform contraction during drying. A similar structural mechanics model, as described by Yang et al. (2001), was used by Curcio and Aversa (2014) to evaluated the effects of potato shrinkage, considering it as a solid-like material, on drying performance by analyzing the spatial distributions of temperature, moisture content, strain, and stress, as a function of operating conditions. They considered the transport phenomena in the drying air for the developed multiphase transport model, predicting the influence of operating conditions on food shrinkage as well as on drying process performance.

Although previously published structural mechanics models dealt satisfactorily with the shrinkage deformation problem, some of these models such as Itaya et al. (1995), Ishiwatari et al. (2012), and Yang et al. (2001) considered linear momentum balance of the materials and used Hooke's law as the constitutive relationship between stress and strain. These are not appropriate for vegetables that deviate from Hooke's law and exhibit visco-elastic characteristics. Therefore in this study, a similar deformation model to that of Sakai et al. (2002), with some modifications, was used for the