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Assessing the potential of using chaotic advection flow for thermal food processing in heating tubes

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ABSTRACT

Most food materials tend to be viscous and in general flow in the laminar regime. In continuous food sterilisation, the non-uniform velocity profile which characterises viscous flow coupled with a non-uniform temperature distribution result in a wide variation of product sterility and nutritional quality across the tube. The challenge is to be able to sterilise the fastest parts in the core region of the tube without over-processing too much the slowest parts near the wall. Chaotic advection is an alternative to turbulence, and uses the stretching and folding property of chaotic flows to promote fluid mixing at low Reynolds numbers. The use of inline static mixers or vortex generators to promote radial mixing and, thus, heat transfer and temperature uniformity, generates large pressure drops but more importantly these devices are unhygienic. We use a validated Computational Fluid Dynamics (CFD) model to show that mechanical vibration is an effective source of chaotic advection. The superimposition of a transverse harmonic motion on the flow of a single-phase viscous fluid in a heating tube, leads to large improvements in thermal processing uniformity and efficiency compared with a conventional process with or without an inline static mixer fitted. Results show that high levels of sterility, processing uniformity and product quality can be achieved in relatively short heating tubes, thus, potentially obviating the need for a holding stage.

1. Introduction

Radial heat transfer in laminar tube flow is governed by slow conduction which leads to a wide radial temperature distribution that poses a considerable challenge in many manufacturing processes. In continuous food sterilisation the non-uniform velocity profile which characterises viscous flow coupled with a non-uniform temperature distribution means that the coldest parts of the fluid at the centre of the tube travel the fastest, thus, resulting in a wide variation of product sterility and nutritional quality across the tube. The challenge is to be able to sterilise the fastest parts in the core region of the tube without over-processing too much the slowest parts near the wall. Increasing the temperature of the inner regions of the fluid is highly desirable so that ideally all parts of the fluid receive equal thermal treatment. Furthermore, better uniformity in the temperature profile helps reduce local variations in the fluid rheological properties which cause distortions in the velocity profile, thus making the flow behaviour of the fluid more predictable. To improve the uniformity of the temperature distribution, methods of increasing radial mixing are required. This problem has been recognised for a long time but effective technological solutions are still missing (Jung and Fryer, 1999).

Radial mixing can be achieved by turbulent flow conditions but the usually high fluid viscosities encountered in practice often make this proposition impractical and/or uneconomical. Alternatively, the use of inline static mixers (Hobbs and Muzzio, 1997; Saatdjian et al., 2012) or vortex generators (Chagny et al., 2000) is prohibited in hygienic processes because of the risk of contamination for their complex geometries promote fouling and make them difficult to clean. A considerable number of studies have demonstrated the effects of pulsating flow or mechanical oscillation on the heat flux and Nusselt number in tube flows (Klaczak, 1997; Gundogdu and Carpinlioglu, 1999; Lee and Chang, 2003). However, the effects on the radial temperature distribution and the development of the thermal boundary layer in a tube have not been addressed in these works.

Research has shown that mixing in non-turbulent flows can be greatly enhanced by complicated particle behaviour caused by chaotic advection. Chaotic advection is a concept derived from

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nonlinear dynamics and is widely used as an approach to investigate transport and mixing problems in fluid flows (Aref, 1984, 1990; Ottino, 1989). In applications where one wants to maximise the rate of mixing of flows, advection is used to accelerate the molecular diffusion process. The classical way to achieve this is through turbulence by using high Reynolds numbers to instigate the formation of a Kolmogorov energy cascade from large to small eddy scales, which results in small-scale structures that lead to rapid molecular diffusion and flow homogenisation. Chaotic advection affords a different mechanism to generate small-scale structures by exploiting the stretching and folding property of chaotic flows whose Lagrangian dynamics quickly evolves into a complex flow pattern; a positive maximum Lyapunov exponent is usually taken as an indication that the system is chaotic. Mixing by chaotic advection is a purely kinematic process which does not require high Reynolds numbers. It has the advantages over turbulence that it does not require the high energy inputs needed to maintain the Kolmogorov cascade in turbulent mixing and can, thus, be exploited in situations where high Reynolds numbers cannot be used. Mechanical vibration is an effective mechanism by which such chaotic advection can be introduced in viscous flow (Eesa and Barigou, 2011; Tian and Barigou, 2015). In this study, we use a validated Computational Fluid Dynamics (CFD) model to demonstrate the large positive effects that a superimposed transverse harmonic motion can have on the extent and uniformity of heat treatment in single-phase laminar tube flow, and the potential benefits it can have for continuous in-flow sterilisation of viscous food fluids.

2. Theory

2.1. Temperature-dependent fluid viscosity model

The single-phase fluid used is an incompressible, temperature-dependent Newtonian fluid whose viscosity is assumed constant at a given temperature and is described by the well-known Arrhenius relationship:

$$\mu = k_0 exp\left(\frac{E_a}{R_g T}\right) \tag{1}$$

where k_0 is a pre-exponential factor, R_g is the ideal gas constant, T is temperature and E_a is the activation energy for viscosity. The constants k_0 and E_a are determined experimentally and their values for various fluids have been reported in the literature (e.g. Steffe, 1996). These parameters, as well as other physical properties (density ρ, specific heat capacity C_p, and thermal conductivity λ) were assumed constant and their values are given in Table 1.

2.2. Transverse harmonic motion

In its basic form (**VF**), the technique uses transverse mechanical oscillations imposed on the tube wall in a direction perpendicular to the tube axis, as illustrated in Fig. 1(*a*), and the wall displacement x is described by the harmonic function:

$$x = A\sin(\omega t) \tag{2}$$

where A is the amplitude of vibration, t is time, and ω is the angular function of the frequency of vibration, f, such that $\omega = 2\pi f$. The linear transversal velocity of the tube wall is then:

$$u = \frac{\mathrm{d}x}{\mathrm{d}t} = A\omega\cos(\omega t) \tag{3}$$

In the new enhanced form of the technique (**VF-SR**), the tube is continuously oscillated transversally but the orientation of oscillation is rotated instantly in a stepwise manner by an angle of 45° about the tube axis, as depicted in Fig. 1(*b*). The time interval, Δt, between change of orientation steps, needs to be optimized for a given set of process conditions. For the conditions considered in this work, a value $\Delta t \sim 10$ s was determined by numerical experimentation, thus, the frequency of the step rotation, Ω, is (and is expected to always be) very low compared with the frequency of lateral oscillations; for example, in this case $\Omega = 0.1$ Hz compared to $f = 50$ Hz. The effects of Ω on the thermal process are further discussed below.

Under steady state, the flow regime was always laminar with a Reynolds number ($Re = \rho\overline{w}D/\mu$) within the range 1.4–90, where D is tube diameter and $\overline{w}$ is mean axial velocity. When the tube was vibrated, the vibration Reynolds number ($Re_v = \rho A\omega D/\mu$) was within the range 22–1400; so flow remained laminar under all conditions of flow and temperature.

2.3. Governing equations

The governing transport equations which are the basis of the CFD model can be written in their general form (Bird et al., 1987), thus:

$$\text{Continuity}: \quad \nabla \cdot \boldsymbol{U} = 0 \tag{4}$$

$$\text{Momentum}: \quad \rho\frac{\mathrm{D}\boldsymbol{U}}{\mathrm{D}t} = -\nabla p + \nabla^2\mu\boldsymbol{U} + \rho\boldsymbol{g} \tag{5}$$

$$\text{Energy}: \quad \rho C_p\frac{\mathrm{D}T}{\mathrm{D}t} = \lambda\nabla^2 T + \mu\dot{\gamma}^2 \tag{6}$$

where p is fluid pressure, $\boldsymbol{g}$ is gravitational acceleration, $\boldsymbol{U}$ is the velocity field and $\dot{\gamma}$ is the second invariant of the shear rate tensor, defined as $\dot{\gamma} \equiv \left[\frac{1}{2}\ (\dot{\gamma} : \dot{\gamma})\right]^{\frac{1}{2}}$.

2.4. In-flow sterility and quality

Food sterility and quality levels can be calculated using the standard Eqs. (7) and (8), respectively:

Table 1
Process parameters used in simulations.

L (mm)	D (m)	T_{in} (°C)	T_w (°C)	$\overline{w}$ (m s^{-1})	k_0 (Pa s)	E_a (J mol^{-1})	R_g (J mol^{-1} K^{-1})	ρ (kg m^{-3})	C_p (J kg^{-1} K^{-1})	λ (W m^{-1} K^{-1})	$\mu = k_0\exp\left(\frac{E_a}{R_g T}\right)$ (Pa s)	
											20 °C	140 °C
2400	0.03	20	140	0.04	5.0×10^{-7}	35 000	8.314	998	4180	0.668	0.868	0.0134