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# Analytical solution of simultaneous heat and mass transfer equations during food drying

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ABSTRACT

A rigorous dimensionless analysis of simultaneous heat and mass transfer equations for food drying was developed and simplified for constant properties. From the simplified result, an analytical solution in 1D rectangular coordinate system was obtained. As opposed to Luikov's Equations (LE), the reported solution considers the effect of temperature on interface moisture content. The analytical solution was obtained by Laplace transform and complex inversion integral with space dependent function as initial conditions. The solution behavior compared with some experimental data was detailed, and the potential of the reported solution for the study of interface phenomena and variable mass transfer properties was discussed.



## 1. Introduction

Food drying by convection is commonly accepted as a process in which convective medium (generally air) heats food surface. This heat produces the water evaporation and the excess produces a temperature increase. As consequence heat conduction and water diffusion are generated within food. Taking $h(T_\gamma - T_i)$ as the heat transferred from air drying to food surface, the fraction of this heat consumed in surface water evaporation is $k_c\rho_\gamma(X_{\gamma i} - X_\gamma)\lambda_{wv}$, and the excess transferred within food by conduction is $-\mathbf{n}_{\beta\gamma}\cdot\nabla(k_\beta T_i)$, where $\mathbf{n}_{\beta\gamma}$ is a unit vector normal to food surface in direction from food ($\beta$) to air ($\gamma$). These facts are summarized in,

$$h(T_i - T_\gamma) = -k_c\rho_\gamma(X_{\gamma i} - X_\gamma)\lambda_{wv} - \mathbf{n}_{\beta\gamma}\cdot k_\beta\nabla T_i \quad \text{in} \quad \mathcal{A}_{\beta\gamma} \tag{1}$$

The moisture content gradient produced by water evaporation is then,

$$k_c\rho_\gamma(X_{\gamma i} - X_\gamma) = -\mathbf{n}_{\beta\gamma}\cdot D_\beta\nabla(\rho_\beta X_{\beta i}) \quad \text{in} \quad \mathcal{A}_{\beta\gamma} \tag{2}$$

the heat conduction within food,

$$\frac{\partial(Cp_\beta\rho_\beta T_\beta)}{\partial t} = \nabla\cdot k_\beta\nabla T_\beta \quad \text{in} \quad \mathcal{V}_\beta \tag{3}$$

And the water diffusion within food,

$$\frac{\partial(\rho_\beta X_\beta)}{\partial t} = \nabla\cdot D_\beta\nabla(\rho_\beta X_\beta) \quad \text{in} \quad \mathcal{V}_\beta \tag{4}$$

The equilibrium relation for moisture content between phases $\beta$ and $\gamma$ is the result of sorption characteristics of product (Córdova-Quiroz et al., 1996) and water vapor pressure ($p_w$),

$$X_{\gamma i} = \frac{a_w p_w/p}{1 - a_w p_w/p}\frac{18}{29} \quad a_w = f(X_{\beta i}, T_i) \quad p_w = f(T_i) \tag{5}$$

where the water activity ($a_w$) as function of moisture and temperature is universally known as sorption isotherm.

Eqs. (3) and (4) are a system of two coupled partial differential equations (PDE) organized as a Cauchy type boundary problem, where the boundaries are defined by Eqs. (1) and (2) and the phase equilibrium by Eq. (5). Eqs. (1) and (5) have been extensively used for describe food drying (Balaban and Pigott, 1988; Mulet, 1994; Wang and Brennan, 1995; Ruiz-López et al., 2004), and therefore they will be called FDE.

Analytical solution of complete FDE has not been reported. In order to develop an analytical solution several simplifications have been introduced, such as, constant properties, negligible heat transfer (constant temperature) and linear equilibrium (interface) relation. Under these simplifications, FDE are reduced to a single parabolic PDE (Eq. (4)), similar to heat conduction equation (Eq. (3)). The analytical solution of simplified FDE (Eq. (4)) is commonly attributed to Crank (1956). However, the first reference applied to drying was by Sherwood (1929) who used the analytical solution of analogous heat transfer Eq. (3) in different geometries from

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**Nomenclature**

| | |
|---|---|
| $a$ | thermodynamic activity |
| $\mathcal{A}$ | surface transfer ($m^2$) |
| $Cp$ | heat capacity (J $kg^{-1}$ $K^{-1}$) |
| $Cm$ | moisture capacity (kg water (kg dry comp)$^{-1}$ °$M^{-1}$) |
| $D$ | mass diffusivity ($m^2$ $s^{-1}$) |
| $D_m$ | mass conductivity (kg water °$M^{-1}$ $m^{-1}$ $s^{-1}$) |
| $h$ | heat transfer coefficients (W $m^{-2}$ $K^{-1}$) |
| $l$ | characteristic length for mass or thermal diffusion (m) |
| $k$ | heat conductivity (W m $K^{-1}$) |
| $k_c$ | mass transfer coefficients (m $s^{-1}$) |
| $K_{eqj}$ | averaged equilibrium coefficients |
| $\mathbf{n}$ | Unit vector normal to interface |
| $p$ | pressure (Pa) |
| $t$ | time (s) |
| $T$ | temperature (K or °C) |
| $U$ | moisture potential (°M) |
| $\mathcal{V}$ | bulk volume ($m^3$) |
| $X$ | moisture content (kg water (kg dry comp)$^{-1}$) |
| *Greek symbols* | |
| $\alpha$ | thermal diffusivity ($m^2$ $s^{-1}$) |
| $\lambda$ | latent heat of evaporation (J $kg^{-1}$) |
| $\theta$ | thermographic coefficient ($m^2$ $s^{-1}$) |
| $\rho$ | dry components concentration (kg $m^{-3}$) |
| $\xi$ | any dimensionless coordinate |
| $\omega$ | any thermal of transfer property (kg $m^{-3}$) |
| *Subscripts* | |
| 0 | initial or reference |
| $e$ | at equilibrium |
| $i$ | at interphase |
| $w$ | for water |
| $v$ | for vapor |
| $\beta$ | solid phase |
| $\gamma$ | air phase |
| *Dimensionless numbers* | |
| $Bi$ | Biot number |
| $Bi_m$ | mass Biot number |
| $Ga$ | new dimensionless number |
| $Ko$ | Kossovich number |
| $Lu$ | Luikov number |
| $Pn$ | Posnov number |

the mathematical compendium by Carslaw (1921). These analytical solutions have been universally used to both describe food drying and estimation of average water diffusivity from drying curves since the first half of XX century until now (Sherwood, 1929; Maskan and Fahrettin, 1998; Trujillo et al., 2007; Dissa et al., 2008; Páramo et al., 2010; Liu et al., 2012; Torrez-Irigoyen and Giner, 2014), but the analytical solution of FDE as a couple system remains unsolved.

In order to find an analytical solution of a similar (albeit not equal) system, the Luikov's (LE) equations must be discussed. LE represent the simultaneous heat and mass transfer in porous solids. This system is mathematically expressed as (Liu and Cheng, 1991; Chang and Weng, 2000; Abahri et al., 2011),

$$h(T_i - T_\gamma) = -(1-\varepsilon)k_m(U_i - U_\gamma)\lambda_{wv} - \mathbf{n}_{\beta\gamma} \cdot k_\beta \nabla T_i \quad \text{in} \quad \mathcal{A}_{\beta\gamma} \tag{6}$$

$$k_m(U_i - U) = -\mathbf{n}_{\beta\gamma} \cdot D_m \nabla U_i - \mathbf{n}_{\beta\gamma} \cdot \theta D_m \nabla T_{\beta i} \quad \text{in} \quad \mathcal{A}_{\beta\gamma} \tag{7}$$

$$Cp_\beta \rho_\beta \frac{\partial T_\beta}{\partial t} = k_\beta \nabla \cdot \nabla T_\beta + \varepsilon \lambda \rho_\beta C_m \frac{\partial U}{\partial t} \quad \text{in} \quad \mathcal{V}_\beta \tag{8}$$

$$\rho_\beta C_m \frac{\partial U}{\partial t} = D_m \nabla \cdot \nabla U + \theta D_m \nabla \cdot \nabla T_\beta \quad \text{in} \quad \mathcal{V}_\beta \tag{9}$$

Three main differences can be appreciated with respect to FDE: (1) water evaporation occurs within solid phase (Eqs. (6) and (8)); (2) a thermogradient effect ($\theta$ in Eqs. (7) and (8)) is considered; and (3) LE equations are given in terms of moisture potential ($U$) in units called °M, instead of moisture content ($X_\beta$). The relation between moisture potential ($U$) and moisture content ($X_\beta$) is through a property called moisture capacity ($C_m$) defined as (Pandey et al., 1999),

$$C_m = X_\beta / U \tag{10}$$

As consequence ($D_m$) is not the average effective diffusivity, but a transfer property called moisture potential conduction (Liu and Cheng, 1991).

LE have been analytically solved for conventional geometries (Tripathi et al., 1973; Liu and Cheng, 1991; Pandey et al., 1999, 2000; Chang and Weng, 2000; Abahri et al., 2011) under the assumption of constant thermophysical ($\rho_\beta, C_m, Cp_\beta, \lambda$) and transfer properties ($D_m, \theta, k_\beta, k_m, h$). Therefore, If FDE would be expressed in terms of LE, their analytical solutions could be applied. The differences (1) and (2) do not represent a problem. Although there is no evidence of thermogradient effect or internal evaporation during food drying (Balaban and Pigott, 1988; Mulet, 1994; Wang and Brennan, 1995; Maskan and Fahrettin, 1998; Ruiz-López et al., 2004; Trujillo et al., 2007; Páramo et al., 2010; Dissa et al., 2008; Liu et al., 2012), this situation can be expressed in LE with $\theta = 0$ and $\varepsilon = 0$.

Actually, the problem is in equilibrium relation: Eq. (5) for FDE and Eq. (10) for LE. In order to express FDE in terms of LE, it is necessary to identify the moisture potential ($U$) in a known variable during food drying. There are two possibilities: water chemical potential ($\mu = \mu_0 + RT\ln(a_w)$), or air moisture ($X_\gamma$). In any case, Eq. (10) is not enough to represent the relation when heat and mass transfer exist, because any of the two drying variables suitable to express $U$ are a strong function of temperature as stated in Eq. (5). Under negligible heat transfer, the average partition coefficient deduced by Córdova-Quiroz et al. (1996),

$$X_{\gamma i} = K_{eq} X_{\beta i} \quad \text{with} \quad K_{eq} = \frac{\int_{X_{\beta e}}^{X_{\beta 0}} (X_\gamma / X_\beta) dX_\beta}{X_{\beta 0} - X_{\beta e}} \tag{11}$$

where the integral is numerically calculated with Eq. (5) at constant temperature, is analogous to Eq. (10) if $U = X_\gamma$. Under this convention $C_m = 1/K_{eq}$, $D_m = C_m \rho_\beta D_\beta$ and $k_m = \rho_\gamma k_c$. However, at variable temperature the relation is not valid. When simultaneous heat and mass transfer are considered, Eq. (5) shows that the mass equilibrium relation must be function of moisture and temperature. The simplest way to express such equilibrium relation between phases is with a linear model (Eqs. (10) and (11) are linear relations),

$$X_\gamma = K_{eq0} + K_{eq1} X_\beta + K_{eq2} T \tag{12}$$

situation not contemplated in LE. Then, there is no way to express FDE in terms of LE.

Therefore, in this work a new analytical solution for simultaneous heat and mass transfer equation with constant properties and a linear relation in interface, for food drying was deduced. The procedure includes a rigorous dimensionless analysis of