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# A liquid diffusion model to describe drying of whole bananas using boundary-fitted coordinates

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ABSTRACT

This article uses a liquid diffusion model to describe convective drying of whole bananas. The experiments were carried out with drying air temperatures at 40, 50, 60 and 70 °C. A two-dimensional numerical solution in generalized coordinates of the diffusion equation with boundary condition of the third kind, obtained through the finite volume method, was used to describe the process. A structured non-orthogonal grid was obtained by digitization of the photography of the fruit. In order to determine the thermo-physical properties, an optimizer was coupled with the numerical solution. The model proposed in this article, which includes shrinkage and variable effective moisture diffusivity, well describes drying for all the experimental conditions, and makes it possible to predict the moisture distributions at any given time. The determination coefficients were greater than 0.99970 and the chi-squares were less than $8.33 \times 10^{-4}$.



## 1. Introduction

Liquid diffusion models are used by several authors to describe thin-layer drying of foodstuffs and other products (Lima et al., 2002; Gastón et al., 2002, 2003; Hacihafizoglu et al., 2008; Silva et al., 2009, 2010, 2011, 2012, 2013; Farias et al., 2013). An advantage of diffusion models when compared, for instance, with empirical ones, is that the first group makes it possible not only to simulate the drying kinetics but also to predict the moisture (and temperature) distribution within the product. The knowledge of these distributions is important because it enables to calculate stresses that can damage the product during drying processes. According to Silva et al. (2010), in several researches that use diffusion models, the shape of product considered in the model is approximated to a simple geometry, with the objective of obtaining an analytical solution or a simple numerical solution. With this purpose, the thermo-physical properties are considered constant, as well as the volume, meaning that the shrinkage is neglected (Doymaz, 2005; Hacihafizoglu et al., 2008). For simple geometries such as slabs, cylinders, and spheres with constant thermo-physical properties, simple analytical and numerical solutions are available in several works (Luikov, 1968; Crank, 1992; Silva et al., 2012). When the real geometry is substituted by a simple geometry, the analysis of the moisture (or temperature) distributions cannot be completely accepted, because it cannot provide information in regard to the precise determination of internal strain or crack formation in the product. Thus, in many occasions, moisture diffusion studies must be performed using the real geometry of the product, in order to identify critical regions subjected to damage.

In the literature, few articles consider ellipsoidal geometries for the product under drying process, for which analytical and numerical solutions of the diffusion equation are proposed (Gastón et al., 2002; Li et al., 2004; Wu et al., 2004; Carmo and Lima, 2005; Hacihafizoglu et al., 2008). Wu et al. (2004), for example, proposed a three-dimensional numerical solution for an ellipsoid shape product. In their solution, constant thermo-physical properties were assumed, as well as an orthogonal structured grid. The effect of the shape of rice on the drying kinetics was studied by Hacihafizoglu et al. (2008), using analytical solutions. The researchers considered the grains as sphere, finite cylinder, and prolate spheroid. It was found that a prolate spheroid represents the drying kinetics of rice better than the other shapes. In another research, the effect of the geometry of the product on the effective moisture diffusivity was studied in drying experiments of Prointa-Isla Verde and broom wheat cultivars (Gastón et al., 2003). The authors considered two shapes for the wheat: sphere and ellipsoid. The study concluded that the model predictions of the ellipsoidal geometry were more comparable to the experimental data. In regards to drying of whole bananas, Lima et al. (2002) used the finite volume

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**Nomenclature**

| | |
|---|---|
| $A, B, a, b$ | coefficients of the discretized diffusion equation or fitting parameters |
| $D$ | effective mass diffusivity ($m^2\ s^{-1}$) |
| $E_a$ | activation energy ($J\ mol^{-1}$) |
| $h$ | convective mass transfer coefficient ($m\ s^{-1}$) |
| $J$ | Jacobian of the transformation ($m^{-3}$) |
| $M$ | moisture content ($kg_{water}\ kg^{-1}_{dry\ matter}$, db) |
| $N$ | number of control volumes |
| $r$ | position of nodal point with respect to the $y$-axis (m) |
| $R$ | radius of the banana (m) |
| $T$ | temperature (K) |
| $x, y, z$ | axes of the Cartesian coordinates system |

*Greek symbols*

| | |
|---|---|
| $\phi''$ | flux per unit area ($m\ kg\ kg^{-1}\ s^{-1}$) |
| $\xi, \eta, \gamma$ | axes of the generalized coordinates system |
| $\tau$ | time at the generalized domain (s) |

*Subscripts*

| | |
|---|---|
| $E, e$ | east |
| $N, n$ | north |
| $NE, ne$ | northeast |
| $NW, nw$ | northwest |
| $S, s$ | south |
| $SE, se$ | southeast |
| SW, sw | southwest |
| $W, w$ | west |
| $P$ | nodal point |
| $i, j$ | representative numbers for $\eta$ and $\xi$ lines of the grid in the generalized domain |

*Superscripts*

| | |
|---|---|
| 0 | previous time |
| * | dimensionless variable |

method to describe the process, considering shrinkage and an ellipsoidal configuration for the product. In this work, liquid diffusion model with constant mass diffusivity was employed to describe the process, and a structured orthogonal grid was used to solve the diffusion equation. Silva et al. (2010) also studied drying of whole bananas using the finite volume method applied to an ellipsoidal geometry, but their model considered effective moisture diffusivity as a variable property.

As is well known, orthogonal grids simplify the numerical solution of the diffusion equation due to the elimination of the cross terms, but they are applicable to only few geometric shapes such as ellipsoid. In addition, the generation of orthogonal grid implies in significant differences in the volume of the control volumes, since the lines of the grid are obligated to be orthogonal in every intersection. In spite of the great number of papers available in the literature proposing numerical solutions for specific shapes, only few works were published for arbitrary geometries (Silva et al., 2009, 2010, 2011; Farias et al., 2013).

This article describes drying of whole bananas using a liquid diffusion model that includes: variable effective mass diffusivity, shrinkage and the real geometry of the product, obtained from the photography of the fruit. For this purpose, generalized coordinates are used to numerically solve the diffusion equation in order to apply the obtained solution to the drying problem.

## 2. Materials and methods

### 2.1. Experiments

Ripe banana *Musa acuminata*, subgroup Cavendish cv nanica was acquired from the local market, Campina Grande, Brazil. Several ripe bananas were peeled and selected by their appearance and size, with no evidence of mechanical damage. Experiments were carried out in a convective dryer (Fig. 1) with vertical flux, controller of temperature and controller of air velocity. Fig. 1(a) shows the flux of air, the grid with an area of 40 × 40 $cm^2$ and trays for the samples. Fig. 1(b) highlights the positions of the trays (15 × 15 × 5 $cm^3$) on the grid, which are changed cyclically during the interruptions of the drying process to weigh the bananas. This dryer was constructed by researchers from Federal University of Campina Grande. Three sieves, each one with 1 whole banana, were placed in the convective dryer, with hot air at 40, 50, 60 and 70 °C. For each temperature $T$, care was exercised to select three bananas with the same dimensions (or very close) to minimize size effect on the experimental data. The radius $R$ and height $L$ was measured every instant at which the bananas were removed from the dryer to measure the mass; and the results for the average values are given in Table 1. This table also presents the moisture contents in dry basis (initial $M_i$, final $M_f$ and equilibrium $M_{eq}$) as well as the drying time $t$ (final $t_f$ and equilibrium $t_{eq}$). Drying was described in this article from the instant zero to $t_f$, but the process took place until the mass reached its equilibrium value ($t_{eq}$). During the experiments, moisture content was measured by the gravimetric method. The bananas were weighed at time intervals ranging from 5 min at the beginning of drying to about 4 h at the final part of the process. At the end of each drying, the bananas were removed from the dryer and placed within an oven at 105 °C.

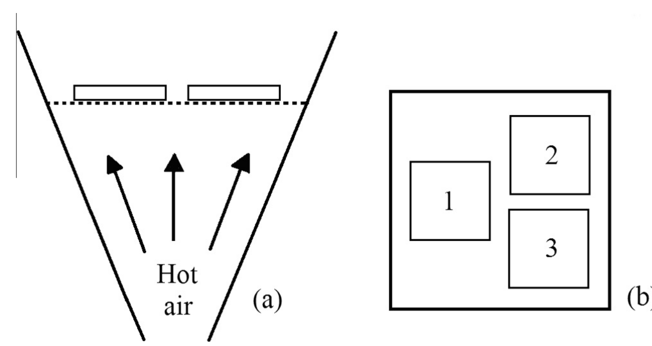


**Fig. 1.** Convective dryer: (a) cross-section and (b) positions of the trays on the cross-section.

**Table 1**
Dimensions, moisture contents and drying times of the bananas for each drying air temperature.

| $T$ (°C) | $\bar{L}$ (mm) | $\bar{R}$ (mm) | $M_0$ (db) | $M_f$ (db) | $t_f$ (min) | $M_{eq}$ (db) | $t_{eq}$ (min) |
|---|---|---|---|---|---|---|---|
| 40 | 179 | 15.7 | 3.22 | 0.407 | 3625 | 0.135 | 5665 |
| 50 | 141 | 14.9 | 3.43 | 0.428 | 2005 | 0.121 | 3805 |
| 60 | 143 | 14.8 | 3.14 | 0.379 | 1420 | 0.108 | 2050 |
| 70 | 165 | 15.3 | 2.90 | 0.349 | 1200 | 0.094 | 1680 |