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Analytical two-dimensional analysis of the transport phenomena occurring during convective drying: Apple slices

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ABSTRACT

The two-dimensional analysis of transport phenomena occurring during drying of a rectangular object is performed employing analytical method. The external flow is predicted numerically and then the convective heat transfer coefficient is determined during convective drying. The convective mass transfer coefficient is obtained through the analogy between the thermal and concentration boundary layers. After calculating average heat and mass transfer coefficients, the heat and mass transfer equations within the object are analytically solved and temperature and moisture distribution is obtained. The present study is contemplated to fill a gap in analytical modeling of two-dimensional heat and mass transfer equations; since most of analytical solutions are one-dimensional, they cannot show the effect of front and rear faces of the moist object on the drying, so the motivation to do this study is to show mentioned effect. Results demonstrated that front and rear faces have significant influences on the drying which should be considered in modeling. It is expected that the model can be applied for other food products and processes involving similar phenomenon. The analytical results are compared to the numerical ones, presenting a reasonable adjustment.

1. Introduction

Drying of fruits and vegetables is a significant part of agricultural industry. The main purpose in drying of fruits and vegetables is the reduction of moisture content to a certain amount, which allows safe storage and preservation. The complexity of drying is because of interrelation between the phenomena of heat and moisture transfer. Therefore, drying is supposed as a complicated and the most difficult procedure in food processing. Many physical and chemical changes occur in food products during the drying process. Many of these are functions of moisture content, temperature and time. Therefore, knowledge of the moisture content and temperature distribution of the product during drying time is needed in order to maintain its stability during storage and improve its final quality.

Modeling of drying processes and kinetics should be determined to control the drying process and it is necessary to choose suitable method of drying for a specific product. The developed models are used for designing new and applicable drying systems as well as selection of desired drying conditions and for accurate prediction of simultaneous heat and mass transfer phenomena during drying process. It also leads to produce the high quality products and save more energy in drying systems.

Different physical, mathematical and numerical methods have been done to describe the drying process. From the reviews by Aversa et al. (2007), Giner et al. (2010) and Barati and Esfahani (2011a), it is clear that rational models are vital to predict the coupled drying and heating rates to enable the prediction of drying curves, drying times, moisture, and temperature history. These models should also be able to predict some insight into the drying mechanism.

An exhaustive analysis is often too difficult in terms of computational time needed to properly analyze the complex transport phenomena involved in food drying (Ruiz-López et al., 2012). Therefore, simplified models have been proposed to capture the drying process of vegetables. In food drying, rigorous analytical solutions are only available for very limited cases under some simplifying assumptions. Córdova-Quiroz et al. (1996) introduced a new model. Their model was capable to reproduce experimental trend of moisture curve of carrot slab. In their model, heat transfer equation was not solved. Hernández et al. (2000) considered the fruits drying process as isothermal, assuming drying temperature equal to air temperature and solving mass transfer equation only. The aim of their work was to show and validate a one-dimensional analytical solution of a mass transfer equation in which shrinkage was taken into account. Pavón-Melendez et al. (2002) proposed a dimensionless investigation for the detailed equations of heat and mass transfer during drying. Their dimensionless analysis makes it possible to predict the behavior of a particular process

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only with an estimation of the scale order of the properties involved. Ruiz-López and Garcia-Alvarado (2007) suggested a model that provides a simple mathematical description for food drying kinetics and considered both shrinkage and moisture dependent diffusivity. They considered constant object temperature during drying. Wu and Irudayaraj (1996) experimentally verified that drying can be actually supposed as an isothermal process just when the Biot number is very low. If Biot number is high, internal transport resistances are also considerable. Barati and Esfahani (2011b) introduced a new model. Their suggested model was able to capture moisture profile within the food while the temperature assumed to be uniform during drying process. In another study, Barati and Esfahani (2012) represented a one-dimensional unsteady state mathematical method to solve coupled heat and mass transfer equations, considering internal resistances to temperature and moisture. Barati and Esfahani (2013) recently presented a reliable one-dimensional model to evaluate simultaneous heat and mass transfer occurring during drying with negligible external resistance to mass transfer. It is worth noting that Barati and Esfahani (2008) proposed an analytical method explaining simultaneous heat and mass transfers involved in cooling of food. They considered heat transfer equation while ignored mass transfer equation and found the results with acceptable accuracy for early times of cooling process.

As seen from the literature, no analytical simulations are carried out to predict the rate of heat and mass transfer within two-dimensional moist objects.The lack of a robust analytical tool for two-dimensional products provides the main motivation for this research because one-dimensional solutions cannot state the significant effect of front and rear faces of object on the drying. The objective of present study is to simulate the convective drying of moist object. In the present work, the external flow and temperature fields have been analyzed with three different inlet velocities. Then, the average convective heat and mass transfer coefficients are obtained. Finally the two-dimensional heat and mass transfer equations are solved using analytical method and the validation of the present model is done with the numerical drying data taken from Kaya et al. (2006).

2. Problem formulation

2.1. Modeling of external flow and temperature fields

The product used for illustration is moist slab cut pieces of apple. With reference to the convective drying process of apple slice in current study: a typical product of apple with the length of 8 centimeter, $L = 8$ cm and height of 2 centimeter, $H = 2$ cm, is considered. Fig. 1 illustrates the problem domain, with its boundary conditions, for the determination of external flow and temperature fields of the drying fluid around the object subject to drying. At the left side, inlet velocities and temperature are $U_\infty = (0.11$ m/s, 0.22 m/s, and 0.33 m/s) and $T_\infty = 323$ K, respectively. Side walls are assumed at U_∞ and T_∞ too, and pressure outlet is considered as outlet condition of flow field. The partial differential equations governing the forced convection motion of a drying fluid in two-dimensional geometry are the mass, momentum and energy conservation equations.

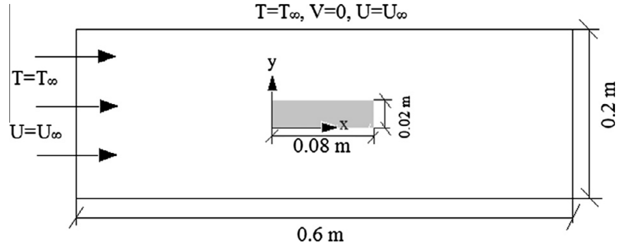

Fig. 1. The problem domain.

The mass conservation equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

The momentum equations are

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

The energy equation is

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

2.2. Modeling of temperature and moisture distribution within the object

The two-dimensional unsteady state mathematical model is developed to simulate the drying process. Temperature and moisture gradient between the food and air are associated with the occurrence of heat and mass transfer. With these purposes, the properties of apple and physical conditions are presented in Table 1 and two assumptions are considered in the solution: (i) moisture content independent thermo-physical properties of the moist object, (ii) negligible shrinkage or deformation of the moist object during drying. The governing two-dimensional heat and mass transfer equations under the above assumptions can be written as

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \tag{5}$$

$$\frac{1}{D}\frac{\partial M}{\partial t} = \frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} \tag{6}$$

The boundary and initial conditions of mentioned equations are Initial conditions:

$$T(x, y, 0) = T_i \tag{7}$$

$$M(x, y, 0) = M_i \tag{8}$$

Left boundary conditions:

$$k\frac{\partial T(0, y, t)}{\partial x} = h(T - T_\infty) \tag{9}$$

$$D\frac{\partial M(0, y, t)}{\partial x} = h_m(M - M_\infty) \tag{10}$$

Right boundary conditions:

$$-k\frac{\partial T(L, y, t)}{\partial x} = h(T - T_\infty) \tag{11}$$

Table 1
The properties of apple and physical conditions (Kaya et al., 2006).

Properties and conditions	
Thermal conductivity, k	0.576 W/(m K)
Density, ρ	856 kg/m^3
Constant pressure specific heat, C_p	1929 J/(kg K)
Moisture diffusivity, D	6.62×10^{-8} m^2/s
Initial moisture content, M_0	7.196 kg/kg of dry air
Initial temperature, T_0	298 K
Moisture content in supplied air, M_∞	0.196 kg/kg of dry air