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A method to estimate anisotropic diffusion coefficients for cylindrical solids: Application to the drying of carrot

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ABSTRACT

In this study, a method to estimate water diffusivity in axial, radial and angular directions during drying of anisotropic cylindrical solids is proposed. The method is based on the analytical solution for non-steady state mass transfer equation in products shaped as longitudinal sections of finite anisotropic cylinders. A sequential approach is applied where radial and axial diffusion coefficients are first estimated from drying curves of whole cylinders with increasing height, while angular diffusivity is obtained thereafter from drying experiments carried out with longitudinal sections of fixed-height cylinders varying cut angle. Developed theory was applied to the analysis of two sets of convective drying data of carrot obtained at 80 °C with an air velocity of 2 m/s. The first experiment set was conducted with carrot cylinders of 2.2 cm diameter and heights of 1, 2, …, 10 cm, while cylinders in the second set were longitudinally cut as halves, quarters, sixths and eighths with a constant height of 10 cm. Under such experimental conditions, identified water diffusivities were in the range of 0.53–2.93 × 10^{-9} m^2/s. Differences in water diffusivity for studied mass transfer directions were significant ($p < 0.05$), with SEM images revealing structural differences between longitudinal and transversal product cuts, thus further supporting numerical results. Present findings suggest that, depending on product, anisotropic diffusion should be included in drying modeling in order to obtain an accurate process description.

1. Introduction

Fickian water diffusion within the product is usually considered the controlling factor during solids drying. In this regard, mathematical modeling of this process very often relies on the assumption of an isotropic solid (Niamnuy et al., 2008; Villa-Corrales et al., 2010; Prakash and Pan, 2012; da Silva et al., 2012, 2013; Lemus-Mondaca et al., 2013; Ruiz-López et al., 2013), even if an anisotropic behavior has been confirmed in several foods such as carrot, green bean, banana, cassava, pumpkin, agave and fish muscle (Ruiz-Cabrera et al., 1997; Roselló et al., 1997; Souraki and Mowla, 2008; Fernando et al., 2011; Gumeta-Chávez et al., 2011; Zhang et al., 2011).

Anisotropy is manifested in water diffusivities differences with respect to mass transfer direction. Therefore, a unique water diffusivity for all directions could potentially lead to misleading predictions for both mean and local moisture content if product geometry or dimensions are changed in subsequent simulation studies. Moreover, anisotropic behavior cannot be anticipated from fiber orientation, as demonstrated by Villa-Corrales et al. (2010) during drying of both longitudinal and transversal mango slabs, thus requiring experimental validation.

Data analysis of anisotropic products imposes certain restrictions or special requirements as opposed to isotropic products. For example, drying simulation obliges mass transfer in more than one direction, increasing computational cost. Therefore, the identification of directional diffusion coefficients is desirable but involves special methodologies as seen in several studies. Ruiz-Cabrera et al. (1997) determined significant differences between axial and radial moisture transport during drying of carrot cylinders eliminating some mass transfer directions by isolating product faces with silicon films in separated experiments. A similar approach was also followed by Roselló et al. (1997) and Souraki and Mowla (2008), where radial diffusivity in green bean was first estimated by preventing mass transfer in axial direction while remaining diffusion coefficient was thereafter identified from experiments with simultaneous radial and axial moisture loss. In a different approach, Fernando et al. (2011) reported a method to estimate axial and radial diffusivities by conducting drying experiments with banana, cassava and pumpkin cylinders of increasing

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Nomenclature

A	series coefficient of the generalized analytical solution for mass transfer equation
c	volumetric concentration of dry solids (kg dry solids/m^3 product)
D	effective diffusivity of water in product (m^2/s)
Fo	Fourier number for mass transfer (dimensionless)
H	absolute humidity of drying air (kg water/kg dry air)
h_m	external mass transfer coefficient (m/s)
J_n	Bessel function of the first kind of order n
K	longitudinal section of cylinder (2 for halves, 4 for quarters, etc.)
L_u	characteristic length for moisture diffusion along the u-axis (m)
$\mathbf{n}$	normal unit vector
r	radial coordinate (m)
R	cylinder radius (m)
t	drying time (s)
u	arbitrary orthogonal coordinate
v	humid volume of drying air (m^3 humid air/kg dry air)
X	moisture content of product (dry basis) (kg water/kg dry solids)
z	axial coordinate perpendicular to xy-plane (m)
Z	half the cylinder height (m)

Greek symbols

α	angular-to-radial diffusivity ratio (dimensionless)
β	axial-to-radial diffusivity ratio (dimensionless)
κ	half the cylinder height-to-radius ratio (dimensionless)
λ	separation constant
μ	separation constant
θ	azimuthal angle in the xy-plane from the x-axis (rad)
θ_{max}	cut angle for cylindrical geometry in the azimuthal coordinate (rad)
ξ	radial coordinate (dimensionless)
ψ, Ψ	dimensionless moisture content of product: local and average, respectively
ζ	axial coordinate (dimensionless)

Subscripts

0	at the beginning of the drying process
e	at equilibrium
i	at the air-product interface
r	denotes the radial direction
z	denotes the axial direction
θ	denotes the angular direction

thicknesses, eliminating the need for insulating product faces. It is important to notice that most studies dealing with anisotropic drying modeling only considered whole cylinders because of the available analytical solutions for radial and axial mass transfer and the reduced complexity of the resulting 2D problem, although numerical solution of drying equations has been also performed to include non-deforming product shrinkage (Roselló et al., 1997). Recently, Adamski and Pakowski (2013) numerically solved 3D heat and mass transfer equations in order to estimate radial, angular and axial diffusivities during pine wood drying with superheated steam systematically isolating faces of whole cylinders as some of the aforementioned references. However, these authors concluded that identification of angular diffusivity require using samples with geometries other than the studied one.

Here, the concept of anisotropic water diffusivities estimation from drying curves obtained at different sample sizes of cylinder-shaped products was generalized to include 3D mass transfer in axial, radial and angular directions. Besides, the 3D analytical solution of mass transfer equation in cylindrical coordinates is used to demonstrate the theoretical relationships between diffusivities in three directions and the effect of a unique diffusivity assumption on drying curves prediction was also calculated.

2. Theoretical development

2.1. Model development

Non-steady state mass transfer by diffusion within an homogeneous and anisotropic material in cylindrical coordinates is given by (Adamski and Pakowski, 2013)

$$\frac{\partial(cX)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD_r\frac{\partial(cX)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(D_\theta\frac{\partial(cX)}{\partial\theta}\right) + \frac{\partial}{\partial z}\left(D_z\frac{\partial(cX)}{\partial z}\right) \quad (1)$$

A convection mechanism is generally used to represent mass transfer from product surface to bulk air (Adamski and Pakowski, 2013; Ruiz-López et al., 2013). Thus, mass flux across boundary is written as

$$-\mathbf{n}\cdot D_r\frac{\partial(cX_i)}{\partial r}\mathbf{e}_r - \mathbf{n}\cdot D_\theta\frac{\partial(cX_i)}{\partial\theta}\mathbf{e}_\theta - \mathbf{n}\cdot D_z\frac{\partial(cX_i)}{\partial z}\mathbf{e}_z = \frac{h_m}{v}(H_i - H) \quad (2)$$

Eqs. (1) and (2) accept the following dimensionless representation,

$$\frac{\partial\psi}{\partial Fo} = \frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial\psi}{\partial\xi}\right) + \alpha\frac{1}{\xi^2}\frac{\partial^2\psi}{\partial\theta^2} + \beta\frac{1}{\kappa^2}\frac{\partial^2\psi}{\partial\zeta^2} \quad (3)$$

$$\psi_i = 0 \quad \text{for} \quad \xi = 0, 1; \quad \theta = 0,\ \theta_{\max}; \quad \zeta = 1 \quad (4)$$

$$\frac{\partial\psi}{\partial\zeta} = 0 \quad \text{for} \quad \zeta = 0 \quad (5)$$

$$\psi = \frac{X - X_e}{X_0 - X_e}; \quad Fo = \frac{D_r t}{R^2}; \quad \xi = \frac{r}{R}; \quad \zeta = \frac{z}{Z} \quad (6)$$

$$\alpha = \frac{D_\theta}{D_r}; \quad \beta = \frac{D_z}{D_r}; \quad \kappa = \frac{Z}{R}; \quad (7)$$

where internal resistance to mass transfer by diffusion was accepted as the only mechanism controlling drying rate (*i.e.*, X_i reaches its equilibrium value X_e instantaneously) and product was assumed to be bounded by three planes parallel to the coordinates axes to produce longitudinal sections of solid finite cylinders. In addition, both constant volumetric concentration of dry solids and constant diffusion coefficients along axes were also considered. Eqs. (3)–(5) may have a separable solution if (i) product has an uniform initial moisture distribution and (ii) shrinkage is neglected. The complete analytical solution to Eqs. (3)–(5) is presented in next section.