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Determination of effective diffusivity and convective mass transfer coefficient for cylindrical solids via analytical solution and inverse method: Application to the drying of rough rice

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ABSTRACT

A method is proposed for the simultaneous determination of the effective diffusivity and convective mass transfer coefficient in solids which can be considered as infinite cylinders. The inverse method was used to fit the analytical solution of the diffusion equation with convective boundary condition to experimental data of thin-layer drying kinetics of products with cylindrical shape. The proposed method was applied to the drying kinetic of rough rice, using experimental data available in the literature. The statistical indicators show that describing the diffusion process with convective boundary condition is more accurate than the description with boundary condition of the first kind, commonly found in the literature.

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1. Introduction

Thin-layer drying of agricultural products depends not only on the product, but also on the type and condition of drying. A specific drying process can be described by an adequate mathematical model as, for example, the liquid diffusion model which involves the diffusion equation (Luikov, 1968; Crank, 1992; Bird et al., 2001). To solve the diffusion equation, the boundary condition at the external surface of the product must be known. Boundary conditions of the first kind have been used for the description of drying with hot air for several types of grains (Gastón et al., 2002; Doymaz and Pala, 2003; Doymaz, 2005; Mohapatra and Rao, 2005; Hacihafizoglu et al., 2008; Silva et al., 2009). However, boundary conditions of the third kind have been found to be more adequate for the drying with hot air for other agricultural products (Queiroz and Nebra, 2001; Wu et al., 2004; Erdogdu, 2005; Mariani et al., 2008).

Generally, the diffusion equation must be numerically solved for solids with arbitrary geometry and, particularly, with variable thermo-physical parameters (Jia et al., 2001; Gastón et al., 2002; Li et al., 2004; Wu et al., 2004; Carmo and Lima, 2005; Silva et al., 2008a). Under certain conditions (spherical or cylindrical geometries, infinite slabs, and constant thermo-physical parameters and volume), the diffusion equation has an analytical solution (Luikov, 1968; Crank, 1992). These solutions are used for the description of thin-layer drying for various agricultural products (Lima et al., 2004; Cunningham et al., 2007; Ruiz-López and García-Alvarado, 2007; Hacihafizoglu et al., 2008).

For the determination of thermo-physical parameters, as effective diffusivity and convective mass transfer coefficient, an adequate mathematical model must be tailored to the description of the drying kinetic of a product. Empirical models, generally simple regressions, can be used to determine the thermo-physical parameters (Park et al., 2002; Tello-Panduro et al., 2004; Silva et al., 2008b). However, in the case of the liquid diffusion model an optimization algorithm, based on the inverse method, can be generally used (Mariani et al., 2008; Silva et al., 2008a, 2009; Da Silva et al., 2009). Mariani et al. (2008) proposed an optimization algorithm for the determination of the apparent thermal diffusivity of banana using a numerical solution of the diffusion equation. Da Silva et al. (2009) proposed two algorithms, one deterministic and another stochastic, to determine the effective mass diffusivity of drying of mushrooms, using the analytical solution of the diffusion equation for an infinite slab with boundary conditions of the first kind. Silva et al. (2009), assuming boundary condition of the first kind, proposed an optimizer which scans the domain of the diffusivity to find the minimum of an objective function. The optimizer was

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Nomenclature

Bi	mass transfer Biot number
D_a	lower limit of the search interval for D_{ef}
D_b	upper limit of the search interval for D_{ef}
D_{ef}	effective diffusivity ($m^2\ s^{-1}$)
h	convective mass transfer coefficient ($m\ s^{-1}$)
J_0	Bessel function of first kind and zero order
J_1	Bessel function of first kind and first order
L	length of the cylinder (m)
n	number of subdivisions of the search intervals
nt	number of first terms of the series before cut-off
N_p	number of experimental points
r	cylindrical coordinate (m)
R	radius of the cylinder (m)
R^2	determination coefficient
S	source term (dimension depends on the process under study)
t	time (s)
V	volume of the cylinder (m^3)
X	moisture content (kg/kg), dry basis
$\overline{X}$	average moisture content (kg/kg), dry basis
X^*	dimensionless moisture content
X_{eq}	equilibrium moisture content (kg/kg), dry basis
X_0	initial moisture content (kg/kg), dry basis
$\overline{X}_i^{exp}$	experimental average moisture content at the ith point
$\overline{X}_i^{ana}$	average moisture content at the same ith point calculated from the analytical solution

Greek symbols

Γ^{Φ}	parameter of the diffusion process (dimension depends on the process under study)
ΔD_{ef}	length of the scanning interval of D_{ef}
λ	parameter of the diffusion process (dimension depends on the process under study)
μ_n	roots of the characteristic equation for an infinite cylinder
σ_i	standard deviation of the experimental average moisture content at the point i
Φ	dependent variable of the diffusion equation (dimension depends on the process under study)
χ^2	chi-square objective function

coupled to the analytical solution of the diffusion equation for a sphere and applied to the drying kinetic of cowpea. The optimizer needs neither an initial value nor the indication of a search interval for the variable of interest.

This article proposes an optimization algorithm coupled to the analytical solution of the diffusion equation with boundary condition of the third kind. The method aims at the determination of the effective diffusivity and the convective mass transfer coefficient of thin-layer drying for products with cylindrical geometry, and was applied to the drying kinetic of rough rice.

2. Material and methods

It was assumed in this article that the liquid diffusion model is adequate to describe thin-layer water transport. This model is widely accepted to describe water transport with boundary condition of the first kind (Doymaz and Pala, 2003; Bello et al., 2004; Mohapatra and Rao, 2005; Thakur and Gupta, 2006), as well as with boundary condition of the third kind (Queiroz and Nebra, 2001; Wu et al., 2004).

2.1. The diffusion equation

The diffusion equation for a property Φ can be written in a general form as (Luikov, 1968; Crank, 1992; Bird et al., 2001)

$$\frac{\partial}{\partial t}(\lambda\Phi) = \nabla\cdot(\Gamma^{\Phi}\nabla\Phi) + S, \tag{1}$$

where λ and Γ^{Φ} are parameters of the diffusion process and S is a source term. For a cylinder with length L, which is much larger than its radius R, a one-dimensional diffusion equation can be applied

$$\frac{\partial}{\partial t}(\lambda\Phi) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\Gamma^{\Phi}\frac{\partial\Phi}{\partial r}\right) + S, \tag{2}$$

where r is the distance of a point from the cylinder axis.

Setting $\lambda = 1$, $\Gamma^{\Phi} = D_{ef}$ (effective diffusivity), $\Phi = X$ (dry basis moisture content) and $S = 0$, Eq. (2) can be rewritten for the moisture transport in a solid considered as an infinite cylinder

$$\frac{\partial X}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\,D_{ef}\frac{\partial X}{\partial r}\right). \tag{3}$$

Eq. (3) can be numerically resolved and, under certain conditions, also analytically. In this article, an analytical solution will be used to describe moisture diffusion in cylindrical bodies.

2.2. Hypotheses for obtaining analytical solutions

The diffusion equation for the description of water transport in solids can be analytically solved under the following hypotheses:

- the solid is homogeneous and isotropic;
- the initial moisture distribution is uniform;
- liquid diffusion is the only transport mechanism of water inside the solid;
- the dimensions of the solid do not vary during diffusion;
- the effective diffusivity do not vary during diffusion;
- the convective mass transfer coefficient is constant during diffusion.

This article studies the drying of cylindrical solids under the assumption that the boundary condition of the third kind is adequate. Therefore, the analytical solution of the diffusion equation for this case is presented below.

2.3. Analytical solution for convective boundary condition

The convective boundary condition, also called boundary condition of the third kind or still Cauchy boundary condition, is expressed by imposing equal internal diffusive flux at the boundary of the infinite cylinder and external convective flux near this boundary, i.e.

$$-D_{ef}\frac{\partial X(r,t)}{\partial r}\bigg|_{r=R} = h\left(X(r,t)|_{r=R} - X_{eq}\right). \tag{4}$$

Here, h is the convective mass transfer coefficient, $X(r,t)$ is the moisture content at radial distance r and time t, X_{eq} is the equilibrium moisture content in a solid of given drying matter, and R is the radius of the infinite cylinder.

The solution $X(r,t)$ of Eq. (3) for an infinite homogeneous cylinder with uniform initial moisture content X_0 and boundary condition defined by Eq. (4) can be obtained by separation of variables (Luikov, 1968; Crank, 1992)

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