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A quick method for thermal diffusivity estimation: Application to several foods

Giampaolo Betta *, Massimiliano Rinaldi, Davide Barbanti, Roberto Massini

Dipartimento di Ingegneria Industriale, Università degli Studi di Parma, Via G. Usberti 181/A, 43100 Parma, Italy

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ABSTRACT

A reliable, quick and userfriendly method for thermal diffusivity estimation has been developed. An appropriate software tool, based on *least square* optimization of a *finite difference* solution of *Fourier's equation*, has been created and an appropriate measuring cell has been designed and made in order to decrease the systematic error in probe positioning. The method has been experimentally validated and its results have been compared with those obtained by three other available methods. Several foods (tomato products, low-acid pasta sauces, olive pate, confectioner's custard and apricot jam) were tested and in every case the method proved to be effective. The developed software also allowed estimation of thermal diffusivity via heat penetration curves obtained by variable temperature treatments. So it has been also possible to exclude the contribution of container material from the estimation of thermal diffusivity of liquid packed foods. The proposed method turns out to be a useful tool for scientific design of several processes, such as sterilization and pasteurization, and for correct control of transport, storage and distribution of foods.

1. Introduction

Correct knowledge of thermal properties is essential for efficient and economical design and control of all food processing operations involving heat transfer such as heating, cooling, freezing, thawing, and frying. Precise and reliable values of thermal properties of foods are necessary to simulate temperature during heat treatments, transport, storage and distribution. Conductive heat exchange is an almost simple physical phenomenon: the classic mathematical model of conduction is Fourier's equation (1)

$$\rho \cdot c_p \cdot \frac{\partial T}{\partial t} = k \cdot \nabla^2 T \tag{1}$$

Conductive heat exchange depends on three physical properties: density (ρ), thermal conductivity (k) and specific heat capacity (c_p). These properties can be included in a single parameter called *thermal diffusivity*, defined by the ratio

$$\alpha = \frac{k}{\rho \cdot c_p} \tag{2}$$

Thermal diffusivity (α) physically relates the ability of a material to conduct heat to the ability to store it. Many methods for determining thermal conductivity and thermal diffusivity were developed. Apart from non-conventional techniques, such as *ac method* (Calzona et al., 1993), *thermal-wave cavity* (Balderas-Lopez and Mandelis, 2001), *thermal lens technique* (Bernal-Alvarado et al., 2003), the majority of available methods were reviewed by Reidy and Rippen (1971) and Choi and Okos (1986). Singh (1992) reported three *models based on food composition* (Dickerson, 1969; Choi and Okos, 1986; Martens, 1980). Information on thermal properties of porous foods is presented in a review paper by Wallapapan et al. (1983). There are two categories of measurement for thermal conductivity and several experimental techniques have been developed for each category; in some techniques, while thermal conductivity is measured, thermal diffusivity is also obtained: (a) *steady-state methods*, such as *hot plate method* (Lentz, 1961), *concentric cylinder* and *concentric sphere method* and (b) *transient state methods* such as *Fitch method* (Fitch, 1935) and *line heat source method* (Nix et al., 1967; Sweat and Haugh, 1974; Nagasaka and Nagashima, 1981; Kumbhar et al., 1981; Choi and Okos, 1983; Rahman and Potluri, 1991; Balaban and Pigott, 1992; Kurozawa et al., 2005); *thermistor probe method* has been used by Valvano et al. (1985), Kravets (1988), van Gelder and Diehl (1996) for the determination of thermal properties, respectively, of biomaterials, milk and tomato products. A reference method was proposed by Ball (1923) and Ball and Olson (1957), who developed what is known as the "*formula*" *method*. It is based on the fact that when heat transfer coefficient of the surrounding medium approaches infinity, the logarithm of the rate of change of temperature becomes

* Corresponding author.
E-mail address: giampaolo.betta@nemo.unipr.it (G. Betta).

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Nomenclature

Greek letters
- α thermal diffusivity
- ψ error function
- ρ density

Latin letters
- c_p specific heat capacity
- H height of the sample
- k thermal conductivity
- N axial space-nodes number
- Q time-intervals number
- R radius of the sample
- S radial space-nodes number
- t time
- T temperature

Super/subscripts
- b imposed at the surface
- e_k experimental at time $(k \cdot \Delta t)$
- i initial
- n axial distance step index
- p time step index
- s radial distance step index
- s_k simulated at time $(k \cdot \Delta t)$

constant in time and space, and is proportional to the thermal diffusivity of the sample. As noted by Mohamed (2003), one of the main serious limitations of this method is that it does not handle the case of variable treatment temperature. Sweat (1986) recommended the calculation of thermal diffusivity by inserting experimental thermal conductivity, specific heat and density values in the thermal diffusivity equation (2). If it is difficult or even impossible to measure directly one or more components in Eq. (2), or if the measured values are not sufficiently reliable and precise,[1] thermal diffusivity can be determined from analytical or numerical solutions of Fourier's equation (1) which fit well the experimental data. In this case thermal diffusivity is estimated as the value of the parameter α which maximizes the quality of approximation of temperature changes in the sample during treatments; a least square algorithm is normally applied to determine the optimal α value. Most recent studies (Carbonera et al., 2004) view numerical simulations of Fourier's equation (1) as the best way to obtain thermal diffusivity value from experimental temperature data. As noted by Markowski et al. (2004), in this case the physical meaning of the thermal diffusivity is different than that based on Eq. (2), and thermal diffusivity determined by that method is usually referred to as *effective* or *apparent* thermal diffusivity. Many authors developed several methods, based on *least square estimation* (LSE), to investigate the thermal properties of foods. Garrote et al. (2000) calculated the thermal diffusivity of potatoes by using an explicit numerical solution. Carciofi et al. (2002) determined the thermal diffusivity of mortadella, cooked in a steam oven, by using actual cooking process data and a least squared algorithm based on an analytical solution of Fourier's equation (1). Zhang et al. (2002) used a finite element method (FEM) for bi-dimensional heat conduction with convective boundary conditions in the precooking and cooling of skipjack tuna (*Katsuwonas pelamis*). Mohamed (2003) exploited a computer solution to calculate the thermal diffusivity value by using a tri-diagonal matrix and an alternative direction implicit finite difference method; experimental validation was carried out by using canned tomato sauce and 8% bentonite suspension. Zorrilla and Singh (2003) used a finite difference method with explicit solution mode to model the heat transfer in double-sided cooking of meat patties considering two-dimensional geometry and radial shrinkage. Carbonera et al. (2004) experimentally determined the thermal diffusivity of a commercial tomato paste by means of both the "*formula*" *method* and an optimization method based on squared error minimisation. Markowski et al. (2004) determined the thermal diffusivity of Lyoner-type sausages during water bath cooking and cooling, using both a numerical and an analytical solution of Fourier's heat transfer equation (1). Kubasek et al. (2006) found out thermal diffusivity of olive oil using a numerical solution based on finite elements. Huang (2007) used a computer simulation program based on finite difference to estimate the apparent thermal diffusivity of beef frankfurters. Mariani et al. (2008) determined thermal diffusivity of banana using a finite difference method coupled to an optimization technique of differential evolution used in inverse method.

The main goal of the present study is to develop and experimentally validate a computer code based on *least square* optimization of a *finite difference* solution of *Fourier's equation* in order to adequately and quickly calculate thermal diffusivity of foods by using heat penetration curves. The second objective is to estimate thermal diffusivity of some food products intended for sterilization or pasteurization for which no references were found.

2. Modelling

2.1. Mathematical model

The assumptions considered in the simulation were as follows: two-dimensional cylindrical sample, homogeneous and isotropic sample, constant thermophysical properties, negligible heat generation inside the sample, infinite heat transfer coefficient at the surface, absence of convective fluxes inside the sample. For a 2D (r,z) axial-symmetric isotropic medium (Fig. 1), the Eq. (1) can be written in cylindrical coordinates

$$\frac{1}{\alpha}\cdot\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2} \tag{3}$$

The following boundary conditions were used:

$$t=0 \rightarrow T(r,z,0)=T_{\mathrm{i}} \tag{4}$$

$$r=0 \rightarrow \frac{\partial T(0,z,t)}{\partial r}=0 \tag{5}$$

$$r=R \rightarrow T(R,z,t)=T_{\mathrm{b}}(t) \tag{6}$$

$$z=0 \rightarrow T(r,0,t)=T_{\mathrm{b}}(t) \tag{7}$$

$$z=H \rightarrow T(r,H,t)=T_{\mathrm{b}}(t) \tag{8}$$

The solution of the above governing equations is difficult to obtain using analytical methods. Therefore, approximate methods of solution are used to solve them.

2.2. Finite difference solution

The method used in the present study is the finite difference approximation. In the finite difference approach, the continuous problem domain is discretized, so that the dependent variables are considered to exist only at discrete points. Derivatives are approximated by differences, resulting in an algebraic representa-

[1] For, e.g. multi-phase or non-homogeneous systems.