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Thermoacoustic transport in supercritical fluids at near-critical and near-pseudo-critical states

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ABSTRACT

Thermoacoustic wave induced transport in carbon dioxide near its critical and pseudo-critical states is investigated numerically. A real-fluid computational fluid dynamic model has been developed considering all of the relevant fluid property variations (including bulk viscosity) near the critical and pseudo-critical states. The predicted results provide interesting details regarding the thermal transport mechanisms at near-critical and pseudo-critical state fluids. As a layer of supercritical fluid (near the critical or the pseudo-critical states) is heated rapidly, the combination of very high thermal compressibilities and vanishingly small thermal diffusivities affect the thermal energy propagation, leading to the formation of acoustic waves as carriers of thermal energy (the so called *piston effect*). The results show that under the same temperature perturbation at the boundary, the induced acoustic field becomes stronger as the critical point or the corresponding pseudo-critical state is approached. The heating rate, at which the boundary temperature is raised, is a key factor in the generation of these acoustic waves. We also study the effect of critically diverging bulk viscosity and different rates of boundary heating on the temperature equilibration mechanism near the critical point. An application of the piston effect in near-critical and near-pseudo-critical fluids for effective thermal transport over a long distance is demonstrated for a supercritical heat pipe.

1. Introduction

When a compressible fluid within an enclosure is subjected to rapid temperature increase at a solid surface, the fluid in the immediate vicinity of the boundary is heated by conduction and tends to expand. However, the sudden expansion of the fluid due to the rapid heating is constrained by the inertia of the unperturbed media and creates a local pressure disturbance, which then leads to the production of a pressure wave. The pressure wave that is generated from the heated boundary travels at approximately the speed of sound within the enclosure and impinges on the opposite wall and then is reflected back. The wave repeatedly traverses between the walls, and its amplitude eventually damps out due to the viscous and thermal losses within the fluid and wall boundary. Thermal and mechanical processes are thus inseparably intertwined in compressible fluids. Rayleigh [1] first approached the idea that a slight variation in temperature can create a mechanical reaction in the form of an acoustic wave in the rest of the fluid. The resulting fluid motion from rapid heating, termed as thermoacoustic convection was numerically investigated by Ozoe et al. [2]. The weak fluid velocities induced by the acoustic wave may significantly enhance heat and mass transfer within the fluid. Consequently, thermal equilibrium in the fluid is reached more rapidly than it would have been by pure conduction. Investigations of the thermally induced acoustic waves are mostly limited to gases [3,4]. However, it was observed that, the thermoacoustic transport phenomena become very efficient near the gas–liquid critical point due to the critical divergence of the thermo-physical properties [5,6].

Near the critical and pseudo-critical states of a pure fluid, the thermo-physical properties exhibit unusual behaviors; showing large gradients for a small change in the state variable (pressure and temperature). Fig. 1 shows the density vs. temperature (ρ–T) relationship of carbon dioxide (critical pressure: 7.377 MPa; critical temperature: 304.13 K; critical density: 467.6 kg/m^3) [7] at different values of pressure. The density of carbon dioxide at a sub-critical pressure (say, 6.0 MPa) varies slightly with temperature, while the density in the near-critical pressure condition (~7.38 MPa) varies widely across the phase interface from the liquid or gas phase to the supercritical fluid phase. Above the critical point, the thermo-physical property variations are mainly characterized by the pseudo-critical states. The pseudo critical state of a pure fluid can be defined as the state in near-critical supercritical region at which the density of the fluid is equal to its critical density ($\rho = \rho_c$, the subscript ‘c’ denotes critical value) and the thermodynamic and

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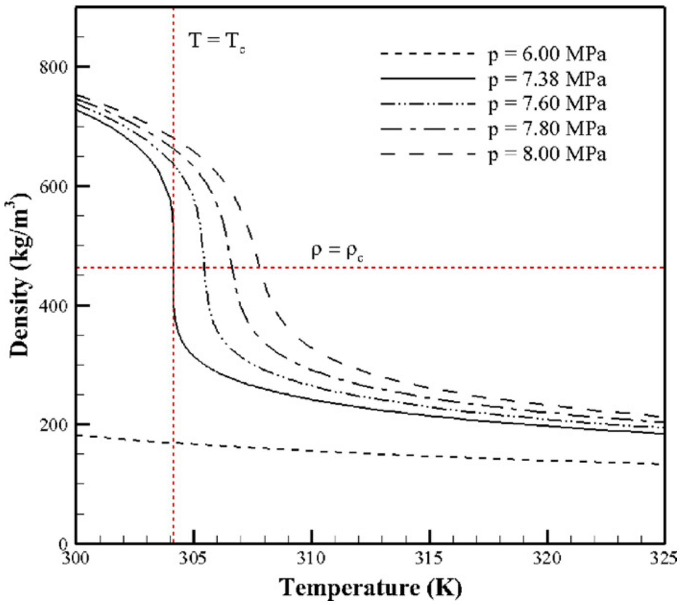

Fig. 1. Variation of density as a function of pressure and temperature for near-critical CO_2 (p_c = 7.377 MPa; T_c = 304.13 K; ρ_c = 467.6 kg/m^3).

transport properties have their maximum rate of change with temperature, at a given supercritical pressure [8]. Its significance is that, below the temperature corresponding to the pseudo-critical state (where $(\rho/\rho_c) > 1$), the fluid has liquid-like properties while above (where $(\rho/\rho_c) < 1$); it more closely resembles a vapor (gas). Apart from these strong dependency on state variables, transport properties of a near-critical fluid undergoing thermoacoustic process can also show dependence on the frequency (f) of the acoustic process [9–11]. Normally, the fluctuations in a fluid relax on a time scale much shorter than the period of the acoustic process; the fluid remains in equilibrium despite the macroscopic motions within and frequency effects are not detected. On the other hand, when a fluid approaches the critical point, the decay time of the fluctuations (τ_{fluc}) becomes very large. Hence a fluid sufficiently close to the critical point can no longer equilibrate within the time scale of the acoustic process and frequency effects become significant. It has been shown in Section 4 that, for the different cases considered in the present study, frequency effect is insignificant and zero frequency (frequency independent) transport properties of near-critical carbon dioxide are used for the calculations.

Fig. 2(a) and (b) shows the variation of thermal diffusivity (α) and thermal conductivity (k) of carbon dioxide with temperature (T) respectively at different pressures and along the critical isochor ($\rho = \rho_c$). It is observed that along the pseudo-critical states, thermo-physical properties of pure fluids experience significant changes. In particular, the thermal diffusivity (α) and acoustic speed (c_s) approaches very small values while the thermal conductivity (k), specific heat (c_p) and isothermal compressibility (β) tends to very high values. Another thermo-physical property, the bulk viscosity ($\mu' = \lambda + (2/3)\mu$, where λ is the second coefficient of viscosity and μ is the shear viscosity) also shows significant variations near the critical point of a fluid. Bulk viscosity is a measure of the resistance of a fluid which is being deformed by a normal stress [12]. It becomes important only for such effects where fluid compressibility is essential. For most gases and incompressible fluids, the Stokes hypothesis ($\lambda = -(2/3)\mu$) [12] is valid and the bulk viscosity (μ') is considered to be negligible. However, for supercritical fluids, near the liquid–vapor critical point, the bulk viscosity is significant and the value diverges at the critical point and along the pseudo-critical states in the near-critical zone. Fig. 2(c) shows the ratio of bulk viscosity to shear viscosity (μ'/μ) of supercritical carbon dioxide as a function of pressure and temperature near the critical point. The bulk viscosity tends to infinity when temperature and pressure approach critical values. This divergence of thermo-physical properties of pure fluids along the pseudo-critical states plays a major role in the thermal equilibration of near-critical fluids.

Thermoacoustic waves in a confined layer of near-critical fluid induce an adiabatic compression of the bulk fluid which results in a homogeneous temperature increase of the bulk. This adiabatic compression known as the '*piston effect*' phenomena in fluids near the gas–liquid critical point, was first explained by Boukari et al. [13] using thermodynamic analysis and was later confirmed by Onuki et al. [14,15] using analytical solution of linearized hydrodynamic equations. Zappoli et al. [16–19] studied the response of a supercritical fluid to a slowly varying temperature disturbance at the boundary using asymptotic expansion techniques. A comprehensive model of the piston effect with a real-fluid equation of state and including the critical divergence of the bulk viscosity was developed by Carles [20]. The mechanisms of heat and mass transport in a side-heated square cavity filled with a near-critical fluid were explored numerically by Zappoli et al. [21] with emphasis on the interplay between buoyancy-driven convection and the thermally induced acoustic waves. The response of a fluid in near-critical conditions to a heat pulse, in the absence of gravity effects was studied experimentally by Garrabos et al. [22]. The study demonstrated that the dynamics of thermal relaxation is governed by two typical time scales, a diffusion time and a time associated to the adiabatic heat transport. Piston effect in a layer of supercritical nitrogen was recently studied numerically by Shen and Zhang [23] by employing a real-fluid equation of state based on thermodynamic relations. However, most of the published research work is based on some simplified (and sometimes questionable) assumptions about the equation of state and thermo-physical properties in the near-critical region. For example, in several studies, thermo-physical properties of supercritical fluid are considered to be constants [24,25] or a function of temperature only [16–20,26] whereas it is a function of both temperature and pressure (or density). The transitional behavior of the thermoacoustic waves from a subcritical to supercritical phase has not been reported in the literature. While piston effect phenomena in near-critical fluids is reported in literature numerous times [13–20,24,27–29], most of these studies are analytical in nature and the piston effect is investigated quite far (Generally, T_c + 1.0 K) from the critical point to avoid critical divergence of the thermo-physical properties. Several earlier studies [14,16] reported that, under the same temperature perturbation at the boundary, stronger acoustic fields can be generated in near-critical fluids as the critical point is approached. But, the different near-critical fluid states considered in these studies were along the critical isochor ($\rho = \rho_c$) only. Thus, a detailed description of the behavior of the piston effect phenomena in the near-critical supercritical region covering the pseudo-critical states is absent in the literature.

In the present study, the generation and propagation of thermally induced acoustic waves in supercritical carbon dioxide (in near-critical and near-pseudo-critical states) are investigated numerically by considering accurate representation of the thermo-physical properties. A high-order (central difference) numerical scheme is used for the simulations. Thermally quiescent and motion free supercritical carbon dioxide in a one-dimensional layer confined by two rigid walls is considered. The NIST database 12 [7] is used to obtain the ρ–p–T relations for supercritical carbon dioxide, along with the static enthalpy $h_0 = f(p, T)$, thermal conductivity $k = f(p, T)$, viscosity $\mu = f(p, T)$ and specific heat $c_p = f(p, T)$ relations. Equations developed by Onuki [30,31] and Moldover et al. [32] are used for the determination of the bulk viscosity (μ') of supercritical carbon dioxide. Different features regarding the flow field