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Dynamic hybrid modeling and simulation of grinding–flotation circuits for the development of control strategies

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ABSTRACT

Process simulation is a very important tool for the design, development, analysis and optimization of technical processes in the mineral industry. The ability to simulate process behavior without the cost of test runs can prevent the loss of man-hours and production, in addition to providing a platform for the development of control tools and strategies. The usefulness of a simulator ultimately relies on how accurately the underlying mathematical model represents real behavior. In mineral processing, due to varied complexities such as strong non-linearities, variable coupling, time varying parameters, etc.; the development of accurate process models becomes an increasingly difficult task.

This paper describes the modeling and simulation of the main components of a concentrator plant, the grinding and flotation circuits. A hybrid dynamic model was favored to better represent the different modes of operation and non-linearities exhibited by the plant. Industrial plant data was used to calibrate the models used in the simulator. Simulation tests show that the performance of the simulator is qualitatively similar to that of a real plant, and it can be effectively used as a tool for the design and simulation of control solutions.

1. Introduction

Copper minerals are too diluted in ore (0.5–2% of Cu) to be smelted directly. Given the great amount of worthless rock, direct heating and smelting would demand too much energy and too much furnace capacity. To address this issue, ores which are destined to pyrometallurgical processing are first concentrated, resulting in a product which contains about 30% of Cu (Biswas and Davenport, 1976). The process of concentration begins with crushing and grinding the mineral to achieve the liberation of the copper contained in the ore. Undergrinding results in a coarse product that may have a low degree of liberation, which will result in diminished recoveries. On the other hand, overgrinding wastes energy and can increase the difficulty of processing valuable minerals. Afterwards, copper minerals are physically separated from non-Cu minerals by froth flotation, resulting in a Cu rich concentrate.

Extensive research has been conducted on the development of mathematical models to represent the processes involved in a concentrator plant, in order to satisfy different objectives such as: plant design and optimization, design of control systems and operator training. Grinding circuits, which are composed of several unit operation such as Semi-Autogenous Grinding (SAG) mills, ball mills, vibratory screens, pebble crushers, sumps, centrifugal pumps and hydrocyclones; have been extensively studied and numerous models have been developed, ranging from static to dynamic models, and phenomenological to empirical models (Austin et al., 1987; Améstica et al., 1993; Salazar et al., 2009; Orellana, 2010; Salazar et al., 2010).

Similarly, several models have been developed to represent flotation cells. Dynamic models based on hydraulic principles used to evaluate control strategies for pulp levels stand out (Stenlund and Medvedev, 2002; Yianatos et al., 2008), as well as models based on mass balances of a set of phases (Pérez-Correa et al., 1998; Casali et al., 2002). In dos Santos et al. (2014), a combination of the compartment model and phenomenological models is presented. The proposed structure is successfully calibrated using experimental data from a pilot plant. Similarly, Newcombe (2014) presents an evaluation of current modeling methods for flotation tested against data gathered from a flash flotation cell. Additionally, a novel method is proposed to interpret and describe the behavior observed, which uses the residence time from within the settling zone of the cell to determine recovery by size at varying heights relative to the mixing zone.

An integration of the grinding and flotation processes, and a development of constrained predictive control strategies for this

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integrated plant is presented in Muñoz and Cipriano (1999). The proposed control strategy is a two-level scheme, with a regulatory level based on linear Model Predictive Control (MPC), and a optimizing level using non-linear dynamic models to maximize economic profit. In Sosa-Blanco et al. (2000), an integrated grinding-flotation simulator for a lead-silver-gold plant, calibrated with real plant data, is presented. The simulator is used to perform economic optimization of the flotation circuit through grinding circuit tuning, increasing the economic efficiency of the plant by 10–20%. Finally, a simulator for a closed-circuit grinding process with flash flotation is presented in Mackinnon et al. (2003). The simulator is developed to estimate the interaction between flash flotation and the grinding circuit and to predict the effect of various changes to the operating conditions on circuit performance. A comparison of the simulator results with operation data from a real plant shows that the simulator closely represents the plant circuit while operating under normal conditions.

In this paper, we propose a methodology to represent, by means of hybrid models, a concentrator plant composed of a grinding circuit followed by a rougher flotation line. Hybrid models incorporate both continuous and discrete variables (meaning that it can only take values from an enumerable set) into the mathematical relations that describe the dynamics of the systems considered. For the rougher flotation circuit, a hybrid model is developed to closely represent the different operating modes exhibited by the cells, using discrete variables to indicate the mode that is currently active. For the grinding circuit, on the other hand, each unit operation is represented by non-hybrid mathematical models, which are interrelated through their input and output streams. These interrelations are represented by discrete variables, which allows the model to increase or decrease the number of processing lines, simulate the blockage of pulp lines, switch between open circuit and closed circuit operation, and so on. An integrated grinding-flotation simulator is essential for an accurate prediction of the copper recovery in response to changes in plant operation, as show in Hatton et al. (2010). Additionally, the hybrid models developed in this paper can be used to design and implement hybrid controllers that take into account the different modes of operation found in concentrator plants.

The organization of this article is as follows. Section 2 offers an introduction to hybrid systems. The model structure, along with the main mathematical equations, are presented in Section 3. Model calibration is described in Section 4. Details of the implementation of the dynamic simulator are presented in Section 5. Validation of the previous models is performed through simulation in Section 6. Conclusions and future work are presented in Section 7. Finally, the complete models and equations used to represent the various processes in the simulator are detailed in Appendix A.

2. Hybrid systems

The model of a system is often associated with differential equations (for continuous-time systems) or difference equations (for discrete-time systems), typically derived from the physical laws governing the dynamics of the systems under study or from empirical observations. Additionally, in many applications the systems under consideration include parts described by logical rules, which allow these systems to transition between different operating modes. The interaction between continuous laws and equations, and logic rules motivates the development of a framework for modeling and controlling these type of systems (Bemporad and Morari, 1999).

A common approach for representing such systems is by using piecewise functions:

$$\begin{array}{l} \mathbf{x}(k+1) = \mathbf{f}_i(\mathbf{x}(k), \mathbf{u}(k)) \\ \mathbf{y}(k) = \mathbf{g}_i(\mathbf{x}(k)) \end{array}, \quad \text{for } d(k) = i, \tag{1}$$

where $\mathbf{x}(k)$ is the state of the system, $\mathbf{u}(k)$ is the input vector and $\mathbf{y}(k)$ is the output vector. $d(k)$ is a discrete variable that represents the mode of operation that is currently active at time-step k. Each subsystems $i \in \{1, 2, \ldots, s\}$ is defined by the (possibly nonlinear) functions $\mathbf{f}_i$ and $\mathbf{g}_i$, and describes the dynamic evolution of the system in the corresponding mode of operation i.

The methodology used in this paper combines non-linear dynamic equations and binary variables to represent the different operating modes of the grinding and rougher flotation circuit. This methodology considers different dynamics for each condition and is suitable to represent non-linear systems, which are very common in mining processes.

3. Model description

As previously stated, the plant studied in this article is constituted by a grinding circuit and a rougher flotation circuit. Both circuits are interconnected through the output stream of the grinding circuit, which serves as the input of the rougher flotation circuit.

The objective of the simulator developed from the mathematical models presented is to be used as a tool for designing and testing different control strategies. As such, the mathematical models used focus on the transient behavior as well as the steady-state behavior. Multiple models are used in both the grinding circuit and the rougher flotation circuit, with the objective of providing a qualitatively close representation of a real concentrator plant. For simplification, the delays introduced by conveyors belts and pipes are not included in the current version of the simulator, although they will be considered in future work for the development of control strategies.

3.1. Grinding circuit

Fig. 1 depicts the grinding plant modeled, which is divided into two stages. In the primary stage, fresh ore coming from a previous crushing stage is fed into a SAG mill, along with the necessary water feed. Then, the mineral pulp from the SAG mill is classified on a vibratory screen. The oversized ore is sent to a pebble crusher, which feeds into the ball mills. The undersized ore, on the other hand, continues to the water sump in the secondary stage. From the sump, the mineral pulp is pumped to two hydrocyclone batteries. The underflow of each hydrocyclone battery is fed into a ball mill, which feeds back into the sump. The final product of this grinding plant is the hydrocyclone overflow, which is fed to the flotation cells.

Various mathematical models are used to represent the different unit operations included in the grinding plant. The following equations, used for the SAG mill, are derived from the models developed in Austin et al. (1987) and Orellana (2010). This dynamic model, based on mass balance, considers two processes operating simultaneously inside the mill: breakage of the mineral particles inside the grinding chamber and classification through the discharge grate. The following equation represents the mass balance used to model the SAG mill:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{w}_{\text{sag}} = \mathbf{f}_{\text{sag}} - \mathbf{p}_{\text{sag}} - \mathbf{K}\mathbf{w}_{\text{sag}} \tag{2}$$

where $\mathbf{w}_{\text{sag}}$ is the hold-up of mineral inside the SAG mill, $\mathbf{f}_{\text{sag}}$ is the mineral feed to the mill and $\mathbf{p}_{\text{sag}}$ is the product of the mill. The lower-triangular matrix $\mathbf{K}$ represents the breakage matrix.

Power consumption in the SAG mill is modeled with the following equation: