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Granular flows in rotating drums: A rheological perspective

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ABSTRACT

A review of granular flow in rotating drums, with a specific focus on the underlying rheology, is presented. The rich coexistence of flow regimes in tumbling mills – the industrial application of rotating drums – highlight the difficulty in obtaining key flow field measurements like velocity and volume concentration distributions, with non-invasive techniques proving the most useful. The mixture of experimentally derived scaling laws underscore the difficulty in defining a suitable granular rheology for tumbling mills. The visco-plastic rheology proposed by Jop et al. (2006) denotes a major step forward in the understanding of dense granular rheology with the scalar form having some experimental corroboration in rotating drums. Unfortunately, the success is militated by the mixed results in subsequent numerical and experimental studies. More specifically, it fails the full tensorial test with notable lack of prediction of the well-known hysteresis between flow initiation and cessation, and the expected phase transition to cataracting flows. Beyond the zeroth order approximation of the visco-plastic rheology, we also explore the pragmatic approaches of depth averaged modelling that include kinetic theory based ingredients to successfully capture the two phase transitions.

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1. Introduction

Particulate flow in rotating drums exhibit complex phenomena such as avalanching, segregation, mixing, drying, aggregation, comminution (abrasion, attrition and fracture) and convection. Industrially, the flow of granular material in rotating drums spans applications of mixing, rotary kilns and comminution. The nature of the latter application (tumbling mills) usually requires high frequency rotations of dense media in drums fitted with radial baffles to facilitate the strain rate and cataracting demands of the process, while mixers and kilns typically require an active flowing layer to mediate the mixing or heat transfer requirements.

Tumbling mills are reported to be <5% efficient in the conversion of input power to useful comminution energy while accounting for more than 60% of the plant's operating costs (Wills, 1997). Current breakage and transport models in tumbling mills are purely empirical machine models. While such models provide highly tweaked recipes for interpolating within the boundary conditions from which they were developed, the ability to extrapolate beyond the window of design is limited, and arguably dangerous, in light of depleting ore bodies and tighter environmental regulations. To remain competitive and economically viable within these stringent and ever changing boundary conditions, mining companies are forced to consider new comminution devices that can potentially increase operational efficiency. Central to this pursuit is an understanding of the mechanisms governing breakage and the rheology underpinning granular flow. While

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the latter is not directly responsible for comminution, accurate constitutive relations between the flow stresses and associated strain rates provide clear indications of damage probability. Recent advances in non-invasive measurement (Parker et al., 1997; Govender et al., 2004) have greatly enhanced the ability to distill scaling laws that govern granular rheology, and ultimately the constitutive relations. On this premise, we expect that a description of granular rheology is more attainable in the near horizon than a fundamentally-based micro-mechanical model of fracture, and therefore, more likely to make a greater impact on comminution practices in the short term.

This review explores the flow regimes within rotating drums, the experimental efforts employed in measuring key rheological ingredients (velocity, volume concentration and flowing layer depth) of these flow regimes, the associated measurement-based scaling laws that necessarily bound the constitutive choices to physical reality, and the recent rheological models that have subsequently emerged. We acknowledge the powerful role that numerical modelling techniques, like the Discrete Element Method (DEM) (Cundall and Strack, 1979; Dury et al., 1998), have to offer; however, we exclude detailed exposition in lieu of the fact that these are well established and validated micro-scale descriptions for predicting bulk flow behaviour. In fact, DEM data is often repackaged into continuum (meso-scale) descriptions for the purpose of testing granular flow models.

2. Flow regimes in rotating drums

Granular flows in rotating drums are often described by a flowing free surface layer over a densely packed rising en-masse that is considered static relative to the rotating drum. Fig. 1 is a simple illustration (not to scale) of the typical flow regimes studied primarily in the physics literature and is based on the schematic given in Midi (2004).

Starting at the top of the surface flowing layer (often referred to as the free surface layer) and moving into the bed along the negative y-direction, Fig. 1, the velocity in the surface flowing layer decreases essentially linearly with depth until very close to the bottom of the flow where the decrease starts to slow down exponentially with further increases in depth. The exponential tail is a solid-like regime that is characterised by dense quasi-static flows in which the deformations are very slow and the particles interact by frictional contact (Roux and Combe, 2002). The linear region is a liquid-like regime that is also densely packed but still able to flow like a liquid with particles interacting by both friction and collision (Pouliquen and Chevoir, 2002; Midi, 2004; Forterre and Pouliquen, 2008). Henein et al. (1983) and Mellmann (2001) classified granular flows in rotating drums by the Froude number $F_r = \frac{\omega^2 R}{g}$; see Fig. 2, where ω is the angular speed in radians per second, R denotes the internal radius of the drum and g is the usual acceleration due to gravity in $[\mathrm{m/s^2}]$. Cascading (Fig. 2d) and cataracting (Fig. 2e) flows exhibit an additional gas-like regime that is characterised by very rapid and dilute flows in which the particles interact mainly by collision (Goldhirsch, 2003). Below the quasi-static regime shown in Fig. 1, the flow is assumed static relative to the rotating drum, i.e. the grains are assumed to be moving with the same angular velocity as the drum, and is often compared to a solid plug moving with the rotating drum – the so-called plug flow.

3. Flow field measurements in rotating drums

Flow field measurements relating to granular rheology are presented herein. The complicated influence of boundary conditions to the flow in rotating drums, especially near the highest and lowest points of the bed, has resulted in most reported measurements occurring along a line similar to the y-axis shown in Fig. 1 (Nakagawa et al., 1993; Yamane et al., 1998; Ding et al., 2001; Orpe and Khakhar, 2001, 2007; Midi, 2004). The choice of this line, which is essentially along the central region of the bed, is motivated by the homogeneity, maximum thickness of the flowing layer and exponential tail, unidirectionality of the flow, and the slow variation of the velocity profile across the layer interfaces.

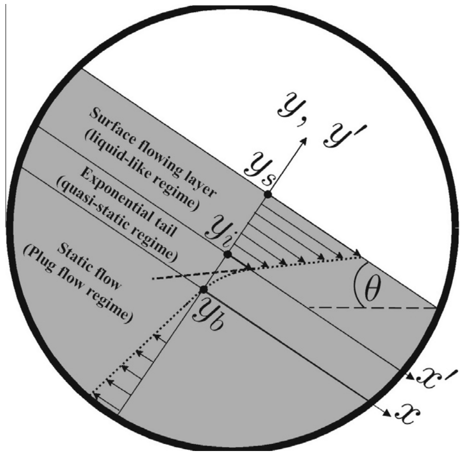

Fig. 1. Granular flow in a clockwise rotating drum according to Midi (2004). The arrows below the top of the surface flowing layer indicate velocities whose profile is delineated by the dotted line. The linear velocity profile in the surface flowing layer is extrapolated along the dashed line to find the intersection with the y-axis.

Many experimental investigations of rotating drum flows have exploited imaging modalities across the electromagnetic spectrum, ranging from transparent end window photographic filming in the visible spectrum (Rogovin and Herbst, 1989; Santomaso et al., 2003) to low wavelength gamma ray techniques, Parker et al. (1997). Particle image velocimetry (PIV) has provided useful measurements of the ensemble-averaged streamwise velocity profiles in the fluidised layer of slowly rotating drum flows, Jain et al. (2002). Nakagawa et al. (1993) used magnetic resonance imaging (MRI) of nearly spherical mustard seeds in a smoothly lined horizontal cylinder to measure the velocity and free surface for speeds operating in the rolling mode; see Fig. 2c. Morrell (1992) employed streak photography of coloured tracers through a transparent end window of a pilot scale tumbling mill to constitute the velocity profile of steel balls. The resulting linear velocity profile formed the key ingredients to his well-known power draught model. Orpe and Khakhar (2001) also employed streakline photography to the flow of mono-sized grains (steel balls, glass beads and sand) in slowly rotating drums and successfully measured the free surface. Positron Emission Particle Tracking (PEPT) has been very successful in studying granular flows in rotating drums that span most of the Froude regimes shown in Fig. 2. Parker et al. (1997) used PEPT in smoothly lined drums operated in the rolling-to-minimally cascading regime to measure a non-linear surface layer and an underlying bed (the rising en-masse region) that deviated from solid body motion due to considerable slip at the drum wall. Ding et al. (2001) overcame the slip problems experienced by Parker et al. (1997) with the use of sand paper (lined along the inner azimuthal wall) to produce measurements consistent with Nakagawa (1994) and Nakagawa et al. (1997). The data also successfully validated their continuum model based on the thin layer approximation.

Govender (2005) used bi-planar X-ray imaging to track the 3D motion of a representative plastic bead moving within a 142 mm diameter experimental tumbling mill containing 6 mm diameter plastic beads and operated in the cataracting flow regime,