Minerals Engineering 77 (2015) 150–158

Contents lists available at ScienceDirect

Minerals Engineering

journal homepage: www.elsevier.com/locate/mineng

Generalizing a shear–volume power model for stirred mill power prediction

Sudarshan Martins [a,*], Peter Radziszewski [a,b]

[a] Department of Mechanical Engineering, McGill University, 3480 University St., Montreal, Quebec H3A 2A7, Canada
[b] Research and Technology Development, Metso Minerals Canada, 795 ave George-V, Lachine, Québec, Canada

ARTICLE INFO

Article history:
Received 23 December 2012
Accepted 13 February 2015
Available online 28 March 2015

Keywords:
Stirred mills
Shear
Power

ABSTRACT

A stirred milling model, based on the concept of shear–volume power, was developed and applied to geometries typically found in mineral processing systems. Generalising this model permits its use to model more complex mill geometries, such as those describing a tapered disk mill and the CoBal mill. Due to the additive property of the shear–volume power, the calculation of the power of complex mills is possible through geometric decomposition. This approach is supported by the calculation of the shear–volume power of a number of simple geometries commonly found in mills, and is used in the calculation of the shear–volume power of a tapered disk mill and the CoBal mill. Finally, by incorporating a breakage model, an equation that offers some usefulness in the design, operation and optimization of stirred mills is presented.

1. Introduction

With ever-decreasing grain sizes, the mineral processing industry has a strong motivation to grind finer, leading to the adoption of stirred milling. In selecting, designing, optimizing and analysing an industrial stirred mill, there is a reliance on computational methods. Techniques such as computational fluid dynamics (CFD) and discrete element modelling (DEM) have provided guidance. However, these methods require significant resources and effort. An easier method has been proposed, with the objective of simplifying the analysis of stirred mills (Radziszewski, 2013). Using this method, the power is expressed as a function of only the viscosity, the mill speed, and the so-called shear–volume.

$$P \cong \mu\omega^2 V_\tau \tag{1}$$

Though the shear–volume power model is simple, it has demonstrated the ability to predict the power draw of three stirred mills configurations: tower, pin and horizontal disk stirred mills (Radziszewski, 2013), validating its use in stirred milling, as illustrated in Fig. 1.

However, the different mills assessed by Radziszewski (2013) had simple geometries, with parallel shear surface pairs, and with one of the surfaces stationary. Here, the main focus of this paper is the generalization of the shear based power model presented by Radziszewski (2013) in order to assess the effect on mill power of more complex geometries, followed by a discussion of different dimensions to future development and use.

2. Expanding the design and analysis space

In the Radziszewski (2013) design space assessment, the geometries which are evaluated are illustrated in Fig. 2. These are fairly simple geometries and are variations of mills found in use in the mining industry. To provide a common basis of comparison, a mill height of 1 m and a shell diameter of 1 m are used throughout the assessment. These dimensions are adopted here.

Looking beyond typical mineral processing applications, a number of more complex stirred mill geometries can be found or devised. These include the angular gap CoBal mill (Orumwense, 1992), and the tapered disk mill, which is presented below. The fundamental design characteristics of the CoBal mill as well as the Tapered mill, adapted to the design parameters used by Radziszewski (2013), are illustrated in Figs. 3 and 4.

In the case of the CoBal mill, the impellor is "W" shaped centered about the shaft. The chamber mirrors the impellor shape and is off-set by some distance providing a constant gap between the impellor surface and the chamber wall.

For the Tapered mill, it is a variation of the disk-on-disk mill illustrated in Fig. 2e, where it is expected that the maximum shear experienced at the impellor disk circumference is greater than that found at the inner circumference of the stationary ring. To

* Corresponding author.
E-mail address: sudarshan.martins@mcgill.ca (S. Martins).

http://dx.doi.org/10.1016/j.mineng.2015.02.011
0892-6875/

Nomenclature

A	Area on the mill over which the shear forces act	r_0	inner radius of surface 1 or offset distance
A_i	*i*th non-intersecting division of the mill area	r_1	outer radius of surface 1
d	distance between shearing surfaces	R_0	inner radius of surface 2
d_1, d_2	distance between shearing surfaces	R_1	outer radius of surface 2
D_b	grinding media diameter	ρ_m	media density
E_B	specific breakage energy	ρ_{sl}	slurry density
F_{80}	feed 80% passing size	T	moment of the shear force
h	height	τ	shear stress
μ	viscosity of the ore	u	velocity
P	shear–volume power	V_τ	shear–volume
P_i	*i*th decomposition of the shear–volume power	ω	angular velocity or angular velocity of surface 1
P_{80}	product 80% passing size	Ω	angular velocity of surface 2
Q	throughput	W_i	bond work index
r	radius of inner surface	x	percentage of the dispersant
R	radius of outer surface		

maintain a constant shear gradient, one can propose the use of tapered disks as shown in Fig. 4. In this case, the shear gradient would need to meet the following criteria:

$$\frac{u}{d} \cong \frac{\omega r_1}{y_1} = \frac{\omega r_2}{y_2} \tag{2}$$

Rearranging gives the gap at the circumference of the stationary ring as:

$$y_1 = \frac{r_1}{r_2} y_2 \tag{3}$$

The dimensions used in the CoBal mill and Tapered mill are found in Figs. 3 and 4 respectively.

3. The shear–volume power—a re-derivation

The simple mill configurations used by Radziszewski (2013) did not capture the full design space of stirred mills. When analysing mills such as the CoBal and the tapered disk mill, a more generalised approach to finding the shear–volume power is needed due to the complex geometry and the different rotational speeds of their components. This approach will be derived here, with some modification to the formulation presented by Radziszewski (2013).

The basis of the shear–volume power is the relationship between the stress and the rate of shearing strain. This is a constitutive equation—it is a phenomenological relationship between two physical quantities that describes the behavior of a class of materials, but is not universal, as illustrated in Fig. 5.

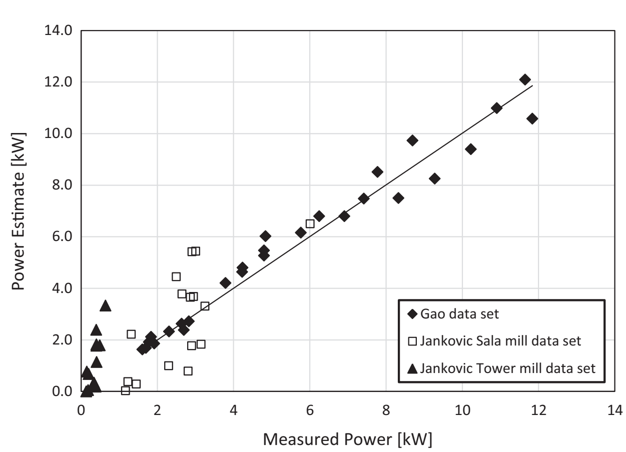

Fig. 1. Stirred mill power comparison (Radziszewski, 2013).

Here, the Newtonian constitutive equation is adopted and is written as

$$\tau = \mu \frac{du}{dy} \tag{4}$$

In cylindrical coordinates, an appropriate coordinate system for stirred mills, the complete Newtonian shear stress is expressed as (Munson et al., 1999)

$$\begin{aligned} \tau_{r\theta} &= \mu\left[r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right] \\ \tau_{\theta z} &= \mu\left[\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right] \\ \tau_{rz} &= \mu\left[\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right] \end{aligned} \tag{5}$$

In keeping with the formulation of shear–volume power analysis, only the dominant shear term is kept (Radziszewski, 2013). All other terms are ignored.

$$\tau \cong \mu\left[r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right)\right] \tag{6}$$

By approximating the derivative by a finite difference, the shear stress becomes

$$\begin{aligned} \tau_1 &\cong \mu\frac{r_1}{d}\left(\frac{u_1}{r_1} - \frac{u_2}{r_2}\right) \\ \tau_2 &\cong \mu\frac{r_2}{d}\left(\frac{u_2}{r_2} - \frac{u_1}{r_1}\right) \end{aligned} \tag{7}$$

Here, it is assumed that the shear is due to two moving surfaces, one at r_1, moving at a speed u_1, the other at r_2, at a speed u_2. The moment of the shearing force on a surface is given by

$$T_{1,2} = \tau_{1,2} A_{1,2} r_{1,2} \tag{8}$$

Since the mill is a rotating system, the shear moment and the angular rate of change define the mill power

$$P = T_1\omega_1 + T_2\omega_2 \tag{9}$$

Without loss of generality, the first terms is examined in more detail. By substitution, this term can be expanded to

$$P_1 \cong \tau_1 A_1 r_1 \omega_1 = \mu\frac{r_1}{d}\left(\frac{u_1}{r_1} - \frac{u_2}{r_2}\right) A_1 r_1 \omega_1 \tag{10}$$

In contrast to the use of Cartesian expression presented in Radziszewski (2013), Eq. (10) reflects the cylindrical nature of the mills. In addition, motion in both limiting surfaces is properly accounted for; when both surfaces rotate at the same speed, this