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Technical note

# The collision efficiency of small bubbles with large particles

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ABSTRACT

Bubble–particle interactions play an important role in many technological processes, e.g., in flotation. Although mineral flotation involves fine particles, this work focuses on the interactions between a small bubble and larger spherical particle and determining their collision efficiency. Based on the theoretical and experimental work, a simple relation for estimating the collision efficiency is proposed. The calculated efficiencies are compared to a large set of experimental data and are found to be in excellent agreement.



## 1. Introduction

Particle–bubble interactions are a central issue in physicochemical hydrodynamics, surface forces, multiphase reactors and the dynamics of wetting films and adsorption at liquid interfaces. The mechanisms of bubble–particle interactions also control the selectivity and efficiency of the flotation process (Dai et al., 1998). Flotation is based on the ability of some solids to remain attached to the bubble surface, and thus, the efficiency expresses the probability that the bubble and particle will make contact and become strongly attached. The basic principles of flotation are currently utilized in many other industrial applications, such as the separation of waste plastics (e.g., Fraunholz, 2004; Pascoe, 2005), where interactions between small bubbles and large particles are considered.

For a bubble to capture a hydrophobic particle efficiently, they must first undergo a sufficiently close encounter. A number of excellent reviews on determining the collision efficiency in mineral flotation mostly consider the gravitational, inertial and interception mechanisms (Nguyen and Schulze, 2004; Dai et al., 2000). The collision efficiency $E_c$ is then given by:

$$E_c = 1 - (1 - E_g)(1 - E_s)(1 - E_i). \tag{1}$$

Here, $E_g$, $E_s$ and $E_i$ denote the efficiencies due to gravity, the interception mechanism and inertia, respectively. When a small bubble – large particle interaction is considered, a similar description can be utilized. The efficiency due to the bubble buoyancy $E_b$ replaces the gravity mechanism, and the $E_c$ relation can be rewritten as:

$$E_c = 1 - (1 - E_b)(1 - E_s)(1 - E_i). \tag{2}$$

If the density of particles is greater than that of the surrounding liquid, particles have a certain settling velocity and therefore their trajectory deviates from fluid streamlines. This deviation may cause particles to collide with the bubble surface (Miettinen et al., 2010). For a Stokes number approaching zero, efficiency due to the gravity mechanism $E_g$ could be expressed by the relationship between the particle velocity $U_p$ and bubble velocity $U_b$ (Flint and Howarth,1971):

$$E_g = \frac{U_p}{U_b + U_p}. \tag{3}$$

Simultaneously for bubbles, the efficiency due to the buoyancy mechanism $E_b$ is defined as:

$$E_b = \frac{U_b}{U_b + U_p}. \tag{4}$$

The collision of particles with the bubble surface by interception is due to a flow which carries particles along the fluid streamlines. The particles come into contact with the bubble surface because of their finite size. For mineral flotation, Sutherland (1948) used this conception to lay the foundations for most of the relations for the collision efficiency. This reasoning can also be used for interactions between small bubbles and large spherical particles, leading to an analogy of Sutherland's expression for the collision efficiency due to interception:

$$E_s = 1 - \frac{8D_p^3}{(2D_p + 2D_b)^3}. \tag{5}$$

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In mineral flotation, the inertial mechanism was often neglected. This mechanism becomes dominant when the Stokes number is greater than unity. In real flotation systems with intermediate Stokes numbers, modern theories such as GSE (Dai et al., 2000) consider both positive and negative inertial forces. For potential flow, less complex models are preferred. Following expression for the inertial efficiency $E_i$ is recommended (Nguyen and Schulze, 2004; Langmuir and Blodgett, 1945):

$$E_i = \left(\frac{St_b}{St_b + 0.5}\right)^2. \tag{6}$$

The Stokes number $St_b$ for a rising bubble near a large settling particle is defined as:

$$St_b = \frac{4}{9}\frac{\rho_l D_b^2 U_p}{\eta_l D_p}. \tag{7}$$

Recently, the collision process between a small rising bubble and larger spherical falling particle was studied experimentally and theoretically (Hubička et al., 2013). The published theoretical model, which is based on an analysis of the forces acting on the bubble, includes a differential equation for the bubble motion and thus allows the bubble trajectory and velocity to be described. In this work, we add new experimental data for a larger particle and we focused on establishing a simple relation for estimating the collision efficiency. The results are compared to a large experimental data-set that includes varying particle sizes, settling velocities, bubble sizes, rising velocities and bubble surface mobility.

## 2. Experimental

Experiments were performed at 25 °C in pure water (de-mineralized, surface tension 71.6 mN/m) and in an aqueous solution of the surface active agent terpineol (Fluka Company, concentration 187 mg/l, surface tension 63 mN/m). The particles were smooth, spherical glass balls with diameters of 14.1 mm and 19.8 mm. The experimental measurements were collected in a glass cell (details in Hubička et al., 2013). Single bubbles were created using a bubble generator. The bubble–particle interactions were observed with a Redlake Motion Pro high-speed camera (500 fps), which was fixed to the translating unit moving parallel with the particle. The particle size, particle velocity (50 and 100 mm/s; controlled by computer), bubble surface mobility (pure and contaminated water) and bubble size ($D_b \sim$ 0.5–0.8 mm) were varied between the different data sets. The images were analysed using NIS-Elements software, by which the bubble size, bubble centre position and collision point were determined. Altogether, 32 sets of experimental data were obtained. For each experimental set, the trajectories from at least ten bubble initial positions were measured.

## 3. Results

The flotation efficiency is defined as the ratio of the number of colliding particles to the total number of particles in the swept volume. In the present case of plastics flotation, the relative sizes of the bubbles and particles are switched. The collision efficiency $E_c$ can be defined analogously as the ratio of the number of bubbles colliding with the particle to the number of bubbles that would collide if their trajectories were not deflected by the flow around the particle. Similar to the case of mineral flotation (e.g., Nguyen and Schulze, 2004), this efficiency can be determined from the bubble grazing trajectory (with initial position $x_{0,g}$), which distinguishes the trajectories of bubbles that encounter the particle surface from those that do not. Based on a simple geometric interpretation of the grazing trajectory, the bubble–particle collision efficiency is:

$$E_c = \left(\frac{2x_{0,g}}{D_p + D_b}\right)^2 \approx \frac{1}{k^2}. \tag{8}$$

The parameter $k$ characterizes the deviation of the bubble from its vertical path, which is primarily due to the liquid flow around the particle. This parameter was measured experimentally. On condition that the frame of reference moves together with the particle and its origin is fixed in the particle's centre, the horizontal position of the bubble centre in its initial position (sufficiently distant from the particle) can be defined as $x_0$ and simultaneously the position of the bubble centre at the collision point can be characterized by its horizontal position, $x_{col}$. Then parameter $k$ is defined as:

$$x_{col} = kx_0. \tag{9}$$

The linear dependence was confirmed both experimentally and theoretically (Hubička et al., 2013). The approximate equality in Eq. (8) can be used if the parameter $k = x_{col}/x_0$ is independent of $x_0$; hence, $x_{0,g} \approx (D_p + D_b)/2k$. This assumption was confirmed for the conditions considered here.

All the experimental results, including $D_p$, $D_b$, $U_p$, $U_b$ and $E_c$, calculated according to Eq. (8) are summarized in Table 1.

In mineral flotation, the collision efficiency is relatively low. Dai et al. (2000) compared all significant theoretical models to the

**Table 1**
Survey of the experimental collision efficiencies $E_c$ with their corresponding particle diameters $D_p$, settling velocities $U_p$, bubble diameters $D_b$ and terminal velocities $U_b$.

| | | Mobile bubble surface | | | Immobile bubble surface | | |
|---|---|---|---|---|---|---|---|
| $D_p$ (mm) | $U_p$ (m/s) | $D_b$ (mm) | $U_b$ (m/s) | $E_c$ | $D_b$ (mm) | $U_b$ (m/s) | $E_c$ |
| 14.1 | 0.050 | 0.500 | 0.105 | 0.71 | 0.553 | 0.064 | 0.60 |
| 14.1 | 0.050 | 0.596 | 0.142 | 0.78 | 0.618 | 0.073 | 0.64 |
| 14.1 | 0.050 | 0.709 | 0.184 | 0.84 | 0.694 | 0.082 | 0.67 |
| 14.1 | 0.050 | 0.798 | 0.231 | 0.87 | 0.803 | 0.094 | 0.71 |
| 14.1 | 0.100 | 0.515 | 0.110 | 0.56 | 0.473 | 0.054 | 0.39 |
| 14.1 | 0.100 | 0.622 | 0.145 | 0.65 | 0.580 | 0.067 | 0.46 |
| 14.1 | 0.100 | 0.731 | 0.195 | 0.73 | 0.691 | 0.080 | 0.51 |
| 14.1 | 0.100 | 0.806 | 0.233 | 0.77 | 0.785 | 0.092 | 0.54 |
| 19.8 | 0.050 | 0.492 | 0.099 | 0.69 | 0.508 | 0.057 | 0.61 |
| 19.8 | 0.050 | 0.640 | 0.151 | 0.74 | 0.617 | 0.072 | 0.65 |
| 19.8 | 0.050 | 0.689 | 0.172 | 0.77 | 0.681 | 0.079 | 0.66 |
| 19.8 | 0.050 | 0.818 | 0.219 | 0.81 | 0.748 | 0.088 | 0.67 |
| 19.8 | 0.100 | 0.479 | 0.098 | 0.51 | 0.494 | 0.059 | 0.45 |
| 19.8 | 0.100 | 0.603 | 0.138 | 0.60 | 0.583 | 0.073 | 0.48 |
| 19.8 | 0.100 | 0.706 | 0.186 | 0.65 | 0.670 | 0.078 | 0.52 |
| 19.8 | 0.100 | 0.805 | 0.231 | 0.69 | 0.741 | 0.087 | 0.53 |