Minerals Engineering 62 (2014) 25–30

Contents lists available at ScienceDirect

Minerals Engineering

journal homepage: www.elsevier.com/locate/mineng

The theoretical partition curve of the hydrocyclone

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ARTICLE INFO

Article history:
Available online 26 October 2013

Keywords:
Hydrocyclone
Partition curve
Classification
Fish-hook
Fine particle separation

ABSTRACT

In many cases, the hydrocyclone partition curve exhibits a non-monotonic course in the fine particle range. The so-called fish-hook effect indicates an increased separation of the fine fraction, which is of practical interest and has a positive effect on solid/liquid separation. However, for classification purposes, the separation is less distinct. In this contribution an equation of a partition curve containing a fish-hook is derived considering the laws of disturbed settling in dense, polydisperse suspensions. The following effects are considered: the entrainment of fine particles in the boundary layer of the coarse settling particles, the hindered settling due to the increased effective density and viscosity of the fluid, and the counter flow of the displaced fluid caused by the settling particles. The calculations indicate that the fish-hook effect is primarily caused by fine particle entrainment, which is influenced by the feed solid content and the feed particle size distribution. An approximated analytical solution for the partition curve is presented for aRosin–Rammler–Sperling–Bennet (RRSB)-distributed feed. Experiments using 25-mm hydrocyclone confirm the calculations.

1. Introduction

The fundamental scheme for the hydrocyclone is shown in Fig. 1a.

The partition curve (Fig. 1b) used to characterize the separation efficiency of the hydrocyclones involves the mass fraction $T(d)$ for each particle size d, which is discharged in the coarse product (underflow). Schubert and Neesse (1980) demonstrated that the typical S-shaped partition curve derives from the superposition of the settling flow and a turbulent diffusion flow in the rotating fluid.

The so-called tapping model (Neesse et al. (1991), Schubert (2010)), which neglects the distribution of the hydrodynamic characteristics in the processing zone of the apparatus, describes the influence of various factors on the separation characteristics. The theoretical partition curve calculated using the free settling velocity according to the Stokes formula, increases monotonically with d (see the dashed curve in Fig. 1b).

However, in many cases in the fine particle range, an increased particle removal can be observed (see the continuous curve in Fig. 1b). This so-called fish-hook effect is subject of many investigations and discussions.

The phenomenon of increased fine particle removal through the underflow leads to practical consequences. For example, increasing the removal of fine particles is beneficial for water purification by removing mechanical impurities. However, the fish-hook effect is detrimental to fine particle classification because it reduces the separation sharpness.

Although the non-monotonic separation function was described in the scientific literature (Finch (1983), many years ago, no consensus has developed regarding the physical basis of this phenomenon.

Some researchers remain skeptical (Flintoff et al. (1987), Nageswararao (2000)) of this effect, believing that it has no physical basis and that the experimental observations are the result of agglomeration phenomena, measurement errors, or the variations in the particles size fractions relative to their shape and density.

These doubts have been analyzed and refuted by Dueck et al. (2007).

After analyzing the statistical properties of the measurements, Bourgeois and Majumder (2013) came to the same conclusion that "the fishhook effect is a real physical phenomenon".

In several publications by Finch (1983), Del Villar and Finch (1992), and Kraipech et al. (2002), empirical correlations have been developed to describe the fish-hook effect.

Schubert (2003, 2004) provided a qualitative explanation of the fish-hook effect using the buoyancy acting on the particles in a non-uniform rotational flow. The random motion of particles of

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http://dx.doi.org/10.1016/j.mineng.2013.10.004

Nomenclature

a	centrifugal acceleration (–)		n	parameter of the distribution function (––
$ß$	entrainment constant (–)		u_{in}	velocity of the suspension flow in the inlet (m/s)
c_V	total volume solid concentration (–)		Δp	inlet pressure (bar)
d	particle size (μm)		s	internal variable of integration (μm)
d_m	characteristic particle size (μm)		S	volume split (–)
D_c	diameter of the cylindrical portion of the hydrocyclone (mm)		$T(d)$	partition function
D_o	overflow diameter (mm)		$V_h(d)$	hindered settling velocity (m/s)
D_{in}	diameter of the inlet (mm)		$V_S(d)$	settling velocity (m/s)
D_t	coefficient of turbulent diffusion (m/s^2)		$V_{St,j}$	Stokes velocity (m/s)
$D(d)$	deceleration function for the disturbed settling (–)		w_{tan}	maximum tangential velocity (m/s)
$E(d)$	acceleration function (–)		$\dot{W}_o$	suspension throughput at overflow (m^3/s)
$f_e(d)$	entrainment function (–)		$\dot{W}_u$	suspension throughput at underflow (m^3/s)
$g(c_V)$	function of solids content (–)		ρ_f	fluid density (kg/m^3)
$q(d)$	density of the particle size distribution (μm^{-1})		ρ_p	solid density (kg/m^3)
H	depth of the fish-hook (–)		μ_f	fluid viscosity (kg/ms)
			Γ	Gamma function (–)

varying sizes in a turbulent environment was considered by Wang and Yu (2010). Majumder et al. (2003, 2007) attempted to explain the origin of the fish-hook effect using a sudden decrease in the settling velocity of the coarser particles due to the Reynolds number restriction. Roldan-Villasana et al. (1993) introduced the concept that a turbulent dispersion could influence the motion of fine particles.

These concepts have not yet been applied in a systematic calculation to determine which parameters—the hydrocyclone, the particulate material and/or the operating conditions—control the characteristics of the fish-hook effect.

Kraipech et al. (2002) pointed to the mechanism of fine particle entrainment by larger particles, but did not offer an appropriate mathematical model. This was provided by Dueck et al. (2004), who explained the non-monotonic separation curves through the entrainment of fine particles caught in the boundary layer of the coarse, rapidly settled particles. This model is based on experiments of Gerhart et al. (1999) and Kumar et al. (2000) and has already been implemented in the computations of Minkov and Dueck (2012).

By varying several parameters, the computer simulations require considerable effort.

Therefore, this work focuses on the approximated analytical calculation of the separation and should be presented in a convenient form for analytical estimations that consider the collective effects of disturbed settling in a dense polydisperse suspension.

2. Partition function

According to the tapping model of Schubert and Neesse (1980), the partition function $T(d)$ as a function of the particle size d can be expressed as follows:

$$T(d) = \frac{1}{1 + S\exp\left[-\frac{D_c}{2D_t}(V_s(d))\right]}. \quad (1)$$

In this equation, the volume split is represented by $S = \frac{\dot{W}_o}{\dot{W}_u}$ in which $\dot{W}_o$ and $\dot{W}_u$ are the suspensions flows of the overflow and underflow, respectively. The value of S can be determined using empirical formulas (Bradley (1965)).

Furthermore, D_c is the diameter of the cylindrical portion of the hydrocyclone, and D_{in} is the diameter of the inlet.

This model assumes that the turbulent diffusion coefficient D_t of the particle is independent of its size. Thus, the shape of the

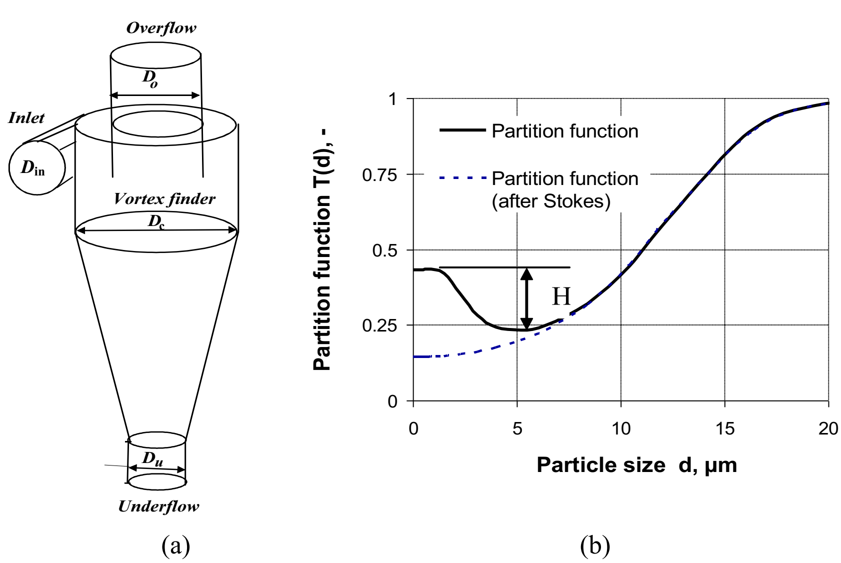

Fig. 1. (a) Principal scheme for the hydrocyclone and (b) partition curve of the hydrocyclone.