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The matrix reduction algorithm for solving separation circuits

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ABSTRACT

Mineral processing circuits typically employ multi-unit staged separation in order to produce a product with sufficient recovery and quality. The design of these separation circuits is largely driven by trial-and-error, incorporating several stages of experimental testing and computer simulation. Linear circuit analysis is a 30 year old fundamental technique that may supplement the more rigorous simulation methods to streamline the formulation of an optimal solution. Unfortunately, linear circuit analysis remains underutilized due to the manual and cumbersome algebra involved. This paper presents a new matrix-based algorithm that allows efficient and automated generation of analytical circuit solutions required for linear circuit analysis. A major advantage of this technique is that it may also be applied numerically to allow simple spreadsheet-based simulation without circular references and iteration. In this communication, the algorithm is thoroughly demonstrated in a trial case, and the analysis is expanded to include 31 simple two and three-unit circuit configurations as well as two especially complex circuits (>15 units). Such large-scale circuit analysis was prohibitively complicated using prior methods. The overall results indicate that while the linear circuit analysis approach is valid, exceptions do exist for many of the original derived design rules.

1. Introduction

The fundamental goal of mineral processing is to increase the value of dilute mining products to a degree that sufficiently justifies the upgrading costs. This goal is achieved by physically concentrating the valuable material through stages of liberation and separation. Ultimately, this beneficiation process generates a tailings product that must be disposed as well as one or more concentrate products that may either be sold in the market or subjected to further downstream refinement. Regardless of the mineral commodity, the physical separations circuit ultimately determines the value of the marketable products in terms of product purity and yield. Typical unit operations for mineral separations include froth flotation, dense-media separators, water-based gravity separators, electrostatic separators, magnetic separators, and optical ore sorters (Wills and Napier-Munn, 2006). Most, if not all, of these separation techniques are not capable of producing a product of sufficient quality and yield with a single unit. As a result, mineral processing plants use staged circuits that incorporate multiple interconnected separation units of different sizes and operational characteristics. These circuits often employ design features that may include series and parallel arrangements, stream splitting, feedback loops (recirculating loads), and multiple feed points. Similar process design approaches also apply to other separation operations, including distillation (e.g. Schweitzer, 1997), plastics recycling (Wolf et al., 2013), algae harvesting (Chen et al., 1998), waste water treatment (Rubio et al., 2002), and juice de-pulping (Araya-Farias et al., 2008). The techniques presented in this paper are derived from generic separation fundamentals and may prove beneficial for any of these industries.

The optimal design of a plant flowsheet is an open-ended, ill-defined engineering problem. The final design must specify the number and type of unit operations, standard operating points, and the stream configuration. Given the large flow volumes, high capital costs, and relative rigidity of the final flowsheet, considerable effort in the design phase must ensure that appropriate separation circuit configuration is selected early in the design process. Over the past 40 years, this design challenge has been assisted by computational modeling and simulation (e.g. Lynch et al., 1981). Since their original inception, process models have grown from low-fidelity empirical curve fits to phenomenological and other physics-based predictive models (King, 2001; Napier-Munn and Lynch, 1992). Today, cheap computation power has permitted the use of computation fluid dynamics (CFD) and discrete element

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method (DEM) simulations throughout the circuit design process. Nevertheless, most commercial circuit design software relies on phenomenological process models for the various unit operations.

One drawback to phenomenological models is that they require extensive experimental testing to calibrate the model parameters. In general, the reliability of the simulation is principally dependent on the size and validity of the experimental data set. Scale-up and greenfield design studies often begin with a nominal flowsheet configuration during bench-scale testing. These results are then subsequently used to build process models and guide locked-cycle and pilot-scale studies; however, the process may demand an alternative flowsheet if the scaled results are insufficient. Depending on the degree of flowsheet alteration, the original bench-scale laboratory data may not be valid for the revised designs. New laboratory tests are required, and the entire process becomes iterative, largely driven by trial-and-error and prior experience (Mendez et al., 2009). Ultimately, the design process may lead to inefficiencies and sub-optimal solutions since the simulation routines do not provide fundamental insight on how the circuit should be configured.

One complementary circuit evaluation and design tool is linear circuit analysis (Meloy, 1983a,b; Williams and Meloy, 1983). This methodology uses partition-based separation fundamentals to analyze the configuration and interconnection of staged separation units while generically considering the actual unit operations. The evaluation of a circuit analysis ultimately provides a single "score" which describes the circuit's inherent separation efficiency relative to other circuits with similar equipment. This score may then be used to rank alternative configurations, while providing fundamental insight on which circuit design features may actually improve performance.

Linear circuit analysis relies on the analytical circuit solution. This mathematically expression describes the overall circuit recovery as a function of the recovery in individual units. Meloy (1983a) has shown that this analytical solution can be derived solely from the unit interconnections using algebraic manipulation. The actual downstream circuit analysis then uses this derived analytical expression to provide a relative measure of the circuit's inherent selectivity. The linear circuit analysis procedure does not rely on strict process models and requires no *a priori* knowledge of the unit operations, the feed material, or the operational environment of the separation. As a result, the methodology is not restricted to specific unit operations or mineral commodities. Rather, the results are universally applicable to all physical separations, whether mineral or non-mineral, even those that do not have a known or well-vetted process model. The limited data requirements make the methodology especially useful in the preliminary plant design phases where extensive feed information and laboratory data may be prohibitively costly or unavailable. A more thorough review of the linear circuit analysis methodology is provided in Section 2.

Apart from the true circuit analysis evaluation, the analytical circuit solution provides other auxiliary benefits, namely in circuit simulation. Many mineral processing simulations (especially spreadsheet-based options) use a sequential, modular solution algorithm. The mass flow rates throughout the circuit are calculated independently and sequentially for each unit. The performance of downstream units is dependent upon the calculations from upstream units. The model calculations are performed unit-by-unit, using the products of upstream units as feed for downstream units. Circulating loads are implemented by invoking iterative calculation (i.e. "Circular References" in Microsoft Excel). The initial value for a circulating load is either set to zero or a nominal "guess" value and the entire circuit is evaluated repeatedly until this value stabilizes within a desired tolerance. With this solution approach, the convergence rate is not only dependent upon the number of units, but also the complexity of the circuit configuration. Alternatively, linear circuit analysis suggests that all processing circuits, despite complexity, have a direct analytical solution. These solutions can be determined by algebraic manipulation and subsequently used to directly calculate the circuit internal and product flows. While iteration may be needed to evaluate the actual process model, iteration is not needed to synthesize the circuit. An example of this synthesis is shown in Section 4.

Despite the scientific appeal of linear circuit analysis, the approach remains largely underutilized. The algebra governing the methodology becomes prohibitively complicated after the circuit surpasses four or five units. Several authors have described procedures for simplifying the cumbersome mathematics associated with circuit reduction (Yingling, 1988; Williams et al., 1992). Unfortunately, these procedures require a working knowledge of advanced probabilistic analysis, flowgraph reduction, and graph theory concepts. Even if these concepts are mastered, the procedures still require manual calculation and, therefore, cannot guarantee a solution devoid of incidental miscalculation.

Addressing these limitations, this paper presents a new method to setup and generate circuit solutions by a matrix reduction algorithm. This procedure may be implemented strictly numerically for the purpose of direct simulation or symbolically to derive the analytical circuit solution. The methodology relies on simple connection matrix setup and manipulation. The procedures are easily scalable to a large range of user-defined circuit configurations, and the solutions are produced quickly and simultaneous for all streams. The remainder of this paper reviews the original mathematical principles of linear circuit analysis (Section 2), describes the steps and requirements of the matrix reduction algorithm (Section 3), shows a calculation example involving the algorithm (Section 4), and finally investigates the complete circuit analysis methodology through application examples (Section 5).

2. Review of linear circuit analysis

2.1. General approach

The linear circuit analysis approach is fundamentally based on separation probabilities. For particulate separations, the concentrate to feed mass flow ratio (C/F) for any particle class is dependent upon the separation technology and the physical properties of the individual particles. Graphically, this ratio may be depicted as a partition function, such as the one shown in Fig. 1. The partition function shows the probability (P) that particles of a given property value will report to the concentrate product. These property values are often normalized against the separation "cut-point" to produce a dimensionless quantity (Z) on the x-axis. Several mathematical formulations of the partition curve for various unit operations are available in the literature (e.g. King, 2001). Ultimately, this curve shows that the dimensionless concentrate to feed ratio (C/F) is a function of the dimensionless property on which the separation is based (Z).

As depicted in Fig. 1, the efficiency of a particular separator can be generically quantified by the slope the partition curve at $Z = 1$. As this slope increases, the shape of the actual partition curve approaches the ideal partition curve. In practice, this value is often quantified by the imperfection or Ecart probable (E_p) value. These indicators are used to rank or compare competing processes or new separation technologies. Nevertheless, a more precise definition of a function's slope is given by the derivative. For a single unit operation ($C/F = P$), the derivative of the functional concentrate to feed ratio at $Z = 1$ is given by:

$$\frac{\delta(C/F)}{\delta(Z)} = \frac{\delta P}{\delta Z} \tag{1}$$