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Numerical investigation of a heuristic methodology for designing precise metal accounting measurement networks

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ABSTRACT

This paper presents a numerical investigation of factors that influence the design of precise measurement networks based on the random error reduction attributes of data reconciliation. The cited factors provide a basis for developing design heuristics that target specified streams for maximal variance reduction. In this investigation, distinct network structures were generated from a case study flowsheet using a strategy that conserves the input and output stream configuration across all generated candidates. The factors were assessed by determining the extent of variance reduction experienced after data reconciliation. Input and output flows were targeted for optimisation owing to their custodial importance in high precision measurement applications such as metal accounting. The results confirm the dependence of variance reduction on measured variance as concluded in previous studies. However, the ratio of stream to parent-node variance emerges as a better predictor of variance reduction for individual streams, particularly in complex network structures.

1. Introduction

Metal accounting is concerned with the estimation of metal from ores and their subsequent distribution in the output streams of minerals beneficiation operations. The 'system of sampling and weight measurement equipment' from which accounting measurements are routinely collected is generally referred to as a measurement network. An important requirement of primary accounting is that boundary measurements are determined with acceptable precisions. There are a numbers of ways of achieving this that include the use of better hardware, duplicate measurements and more stringent sampling and analysis procedures. In secondary accounting, these methods are also applied to all pertinent internal streams. Notwithstanding the improved precisions attained, these measures may consume disproportionate amounts of resources compared to the results achieved. In addition, the expectation of closed balances is often unfulfilled in the short term as a result of random errors that degrade the quality of metal accounting data. Consequently, the minimisation of random error in measurements remains an important objective in applications such as metal accounting.

Steady state data reconciliation is frequently used to adjust measured data so that network constraints are verified while measurement variances are simultaneously reduced. The extent of variance reduction depends on the choice of measurement schemes, and is only determined after data reconciliation is performed on a given network design (Hodouin et al., 1998). The problem of designing networks in order to meet specified objectives based on the measurement correction attributes of data reconciliation has been extensively treated in 'sensor network design' studies (Narasimhan and Jordache, 2000). A drawback of the data reconciliation approach however, is that the extents of variance reductions experienced by individual measurements are generally unpredictable. In other words, one cannot predict which measurement variances will be reduced the most and by what margin of reduction. Thus performing data reconciliation on a given data set may result in a good reduction in variance on streams that have little relevance while leaving the variance of important streams (e.g. boundary streams) relatively unchanged. This detracts from the data reconciliation process the potential of being utilised as a single step design tool for predicting improvements in precision on targeted measurements at the conceptual stages of network design.

Save for a few efforts (Lyman, 2005; Bepswa et al., 2006, 2008; Chakraborty and Deglon, 2008), attempts to develop guidelines or heuristics that predict the reduction in variance through data reconciliation for specific streams had not been explicitly dealt with in the literature. This paper numerically tests design factors in the aforementioned literature that can be used as a basis for developing heuristics/guidelines for designing precise measurement networks through the error reduction attributes of steady-state data reconciliation.

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Nomenclature

Abbreviations
s.t. subject to

Indices
a adjusted value
m measured value
n nodes
s streams
$i(n_1, n_2)$ internal streams connecting nodes n_1 and n_2
T transpose

Labels
n, n_1, n_i, n_j, n_k nodes

Parameters (units)
$\sigma^2_{m(n_1)}$ measured variance of observed stream attached to node n_1 ([mass flow units]^2)
M_n sum of measured variance of all streams attached to node n ([mass flow units]^2)
N_n total number of nodes in a flowsheet (–)
N_s total number of streams in a flowsheet (–)
$Var(measured)$ sum of measured variance of all streams in a flowsheet ([mass flow units]^2)
x_m vector of measured component flow rates (mass flow units)

Variables (units)
$\sigma^2_{a(n_1)}$ adjusted variance of observed stream attached to node n_1 ([mass flow units]^2)
R matrix of observed variance reduction ratios (–)
$Var(adjusted)$ sum of adjusted variance of all streams in a flowsheet ([mass flow units]^2)
x_a vector of adjusted component flow rates (mass flow units)

Matrices
A flowsheet incidence matrix
Σ_m measured variance–covariance matrix
Σ_a adjusted variance–covariance matrix
I identity matrix

1.1. Background

Steady-state data reconciliation is usually formulated as a constrained weighted least-squares adjustment problem. Eqs. (1) and (2) outline the general structure of the problem.

$$Min_{x_a} \quad (x_a - x_m)^T \Sigma_m^{-1} (x_a - x_m) \tag{1}$$

$$s.t. \quad Ax_a = 0 \tag{2}$$

The weights are usually the inverse of the measurement error variances (Σ_m) and the constraints to be satisfied are the mass balance equations that define the measurement network as described by the network incidence matrix A. The adjusted and measured component flow rates are represented by the vectors x_a and x_m respectively. Solving Eqs. (1) and (2) for the adjusted flow rate results in Eq. (3). The reconciled flow rates x_a verify Eq. (2). Estimates of the reconciled flow rate variances can be obtained by using the propagation of variance through formulae to yield Eq. (5), where Σ_a is a matrix containing adjusted flow rate variances.

$$x_a = \left[I - \Sigma_m^{-1} A^T \left(A\Sigma_m A^T\right)^{-1} A\right] x_m = Bx_m \tag{3}$$

where

$$B = \left[I - \Sigma_m^{-1} A^T \left(A\Sigma_m A^T\right)^{-1} A\right] \tag{4}$$

$$\Sigma_a = B\Sigma_m B^T \tag{5}$$

In order to assess the reduction in variance experienced due to steady-state data reconciliation, the adjusted variances can be expressed as a fraction of the corresponding measured variances as shown in Eq. (6) to yield 'variance reduction ratios' for each measured variable in the network.

$$\frac{\Sigma_a}{\Sigma_m} = BB^T \tag{6}$$

$$R = H\frac{\Sigma_a}{\Sigma_m} \tag{7}$$

An extraction matrix (H) can be used to observe selected variance reduction ratios (R) as shown in Eq. (7). In the current study, H is designed to extract variance reduction ratios for the important boundary streams. For clarity, the foregoing derivation assumes the simplified case of a single-component mass flow network where there are no cross-stream correlations between measurement error variances. Hence Σ_m essentially reduces to a diagonal matrix with off-diagonal elements identical to zero.

1.2. Design factors for precise measurement networks

Lyman (2005) provided earlier insights into *a priori* design of measurement networks based on the achievement of overall precision through steady-state data reconciliation. The work produced an explicit expression of the average improvement in precision expected in linear circuits consisting of N_n nodes and N_s streams, in which all streams are measured with equal variances (Eq. (8)).

$$Average\left[\frac{Var(adjusted)}{Var(measured)}\right] = 1 - \frac{N_n}{N_s} \tag{8}$$

Eq. (8) introduces an important decision variable, the circuit node to stream ratio (N_n/N_s), which can be used to compare network designs based on network structure. However, the model only predicts the average variance improvement for the linear network and hence cannot be used to predict precision improvement for individual streams. Later, Chakraborty and Deglon (2008) attempted to resolve this by deriving a 'flowsheet-independent' formula that solved for variance reduction ratios of individual input and output streams (hereon referred to as terminal streams) in conceptual networks by symbolically solving Eq. (6) resulting in

$$\frac{\sigma^2_{a(n_1)}}{\sigma^2_{m(n_1)}} = 1 - \sigma^2_{m(n_1)}\left(\frac{\prod_{\forall n \neq n_1} M_n - t^{numerator}}{\prod^{N_n} M_n - t^{denominator}}\right) \tag{9}$$

Here, the reconciled or adjusted variance $\left(\sigma^2_{a(n_1)}\right)$ of an observed terminal stream (attached to node n_1) is expressed as a fraction of the corresponding measured variance, $\sigma^2_{m(n_1)}$. M_n is the sum of variances of all streams associated with the respective parent node n in the given circuit, N is the total number of nodes in the circuit and the t terms are measures that gather network 'stream effects' on the numerator and denominator expressions of the quotient term in the equation. Significantly, Eq. (9) discerns internal and terminal stream measurements, ostensibly on the basis of their mathematically separable responses to variance reduction through steady-state data reconciliation formulation. Through simulations of