Minerals Engineering 36–38 (2012) 219–230

Contents lists available at SciVerse ScienceDirect

Minerals Engineering

journal homepage: www.elsevier.com/locate/mineng

Variations in the induction period for particle–bubble attachment

David I. Verrelli [a,*], Peter T.L. Koh [b], Warren J. Bruckard [a], M. Philip Schwarz [b]

[a] CSIRO Process Science and Engineering, Bayview Avenue, Clayton, VIC 3168, Australia
[b] CSIRO Mathematics, Informatics and Statistics, Bayview Avenue, Clayton, VIC 3168, Australia

ARTICLE INFO

Article history:
Available online 6 May 2012

Keywords:
Flotation bubbles
Flotation kinetics
Froth flotation
Mineral processing
Modelling

ABSTRACT

The success of flotation is governed by particle–bubble attachment. A convenient means of predicting the attachment compares the duration of sliding 'contact' with the induction time. Whereas a number of theoretical models and empirical correlations exist for evaluating the 'sliding contact' time, comparatively little is known about the key determinants of induction time. It is usual to take the induction time as a constant for a given particle type in a given physicochemical environment. Recent measurements using the CSIRO Milli-Timer suggest considerable variation not just of sliding times, but even of induction times for individual 'ideal' particle fractions. In particular, it is relevant to consider dependence on the approach trajectory of the particle, which can be quantified in terms of the polar angle of first proximal contact of the particle with the bubble. This hypothesis is strongly supported by our numerical modelling, which rigorously describes the microhydrodynamics, and predicts substantial increase in induction period with increasing polar angle of impingement. We also observe the influence of neighbouring particles on the attachment of an approaching particle; these multi-body interactions, which are not considered in the majority of theoretical models, can be sufficient to stimulate attachment.

1. Introduction

Seventy-seven years ago, while industrial flotation was still in its infancy, Sven-Nilsson (1934) proposed the existence of an *Induktionszeit*, or induction period (τ), which constituted the minimum duration of proximal 'contact' between a bubble and a particle in order to achieve attachment. This parameter is still not well understood. In Sven-Nilsson's experiments a bubble was manually pushed against a solid surface. It was soon recognised that where the bubbles and particles move of their own accord in an flotation cell, due to the influences of gravity and the surrounding fluid motions, the nature of the 'contact' process may be different — and more complex — than in those original experiments.

Most frequently a particle will approach a bubble toward a point slightly offset from the apex. As the two objects near each other, the hydrodynamic resistance arising from water escaping from the gap between them will cause some deviation in their trajectories. If the offset is not too large — *i.e.* it is less than the so-called 'collision radius' defined by Sutherland (1948) — then despite the deviation, the particles are still able to come close enough to the bubble to seem practically to touch it, and thenceforth slide along its surface[1]. The duration of the sliding, in the absence of attachment, is $\Delta t_{\mathrm{slide}}$. When the offset is exactly equal to the collision radius, then the particle will just 'graze' the bubble (Nguyen and Schulze, 2004, pp. 185ff), and its trajectory is described as the 'limiting trajectory' (Derjaguin et al., 1984, p. 80). See Fig. 1; the pathlines shown in this figure are purely schematic. Actual particle pathlines are presented in our previous publications (Verrelli and Koh, 2010; Verrelli et al., 2011).

Experiments carried out by several workers have reproduced this manner of interaction in a controlled environment so as to be able to estimate sliding durations (Bogdanov and Filanovsky, 1940; Schulze and Gottschalk, 1981a, 1981b; Schulze, 1992; Wang et al., 2003), particle trajectories and velocities (Flint and Howarth, 1971; Schulze and Gottschalk, 1981a, 1981b; Wang et al., 2003; Nguyen and Evans, 2004b; Verrelli and Koh, 2010; Verrelli et al., 2011), attachment efficiencies (Wang et al., 2003), and induction times (Verrelli and Koh, 2010; Verrelli et al., 2011) by direct observation.

* Corresponding author. Address: Box 312, Clayton South, VIC 3169, Australia. Tel.: +61 3 9545 8689; fax: +61 3 9562 8919.

E-mail addresses: David.Verrelli@csiro.au (D.I. Verrelli), Peter.Koh@csiro.au (P.T.L. Koh), Warren.Bruckard@csiro.au (W.J. Bruckard), Phil.Schwarz@csiro.au (M.P. Schwarz).

[1] For very small offsets it is possible for attachment to occur in a slightly different fashion, without sliding, attributed to 'impact' upon collision (Schulze, 1984, p. 70; Nguyen and Schulze, 2004, pp. 257ff.). It has been suggested that sliding collisions are "considerably more effective" in achieving attachment than 'impact' collisions (Schulze, 1992). 'Impact' collisions may be negligible in column flotation (Schulze, 1992). While they may be of importance in impeller flotation machines (Schulze, 1992), the proposed theories do not yet have predictive power (Nguyen and Schulze, 2004, p. 282). Such interactions are not considered in the present investigation.

0892-6875/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.mineng.2012.03.034

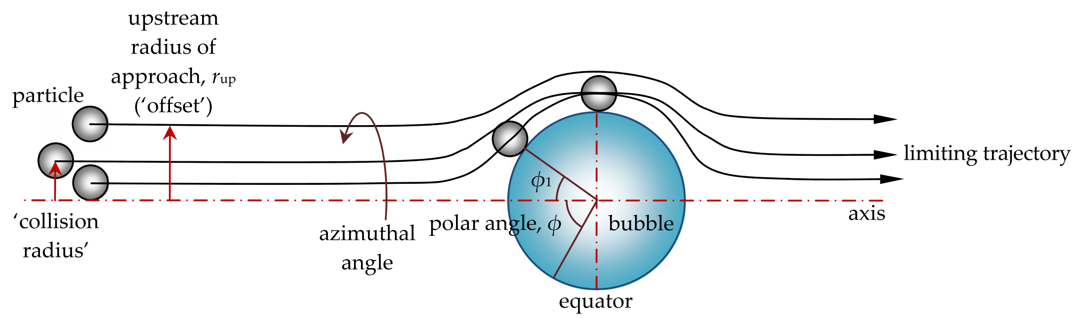

Fig. 1. Illustration of a few parameters relevant to particle–bubble interaction. The trajectories illustrated follow the particle centres and are thus 'pathlines', rather than fluid 'streamlines'.

By comparison of the (potential) sliding duration[2] with the induction period, a prediction can be made as to whether or not attachment occurs. The simplest option is to set a sharp cut-off, with all cases in which

$$\Delta t_{\text{slide}} \geq \tau$$

predicted to yield attachment. It is also possible to employ a more probabilistic approach, in which the likelihood of attachment might be specified to depend upon, say, the amount by which Δt_{slide} exceeds τ, allowing for stochastic factors (Sven-Nilsson, 1934; Schulze, 1992; Wang et al., 2003; Nguyen and Schulze, 2004, p. 190f.). For clarity we will discuss only the former approach; however, our analysis applies equally well in the latter instance.

Beginning with Sutherland (1948), a lot of effort has gone into modelling the trajectories of particles interacting with a bubble. Broadly speaking, these can be classified as either analytical or numerical models. In the former case, an analytical equation directly describes the sliding time (or encounter efficiency, *etc.*) in terms of system parameters that are known at the outset, *e.g.* particle size. In the latter case fundamental equations are written to describe the individual forces as functions of the state of the system, which are not known *a priori*, such as the tangential velocity of the particle with respect to the bubble at a given moment in time. To obtain useful results, such as Δt_{slide}, the system of equations has to be solved numerically. Well-known analytical models are reviewed in standard texts (*e.g.* Nguyen and Schulze, 2004, pp. 265ff.) (see also Dai et al., 2000). Examples of numerical models are those presented by Schulze (1992), Liu and Schwarz (2009a, 2009b), and Verrelli et al. (2011). While the analytical models can be appealingly intuitive and arguably easier to use, they cannot provide the same level of accuracy and detail available in properly constituted numerical models (*e.g.* Crank, 1975, p. 137).

All of these models agree that Δt_{slide} depends on the upstream radius of approach, r_{up}, or, equivalently, upon the polar angle of sliding commencement, φ_1. Thus, for a given particle–bubble pair under a given flow scenario a range of sliding periods can be realised[3]. Moreover, the experimental observations indicate a significant degree of scatter in the estimated sliding times for even a fixed value of φ_1 (Schulze and Gottschalk, 1981a, 1981b).

In contrast, the induction period seems universally to be treated as a constant for a given particle–bubble pair under a given flow scenario (in a given physicochemical environment). The particle trajectories in the vicinity of the bubble do vary substantially as a function of r_{up}, and φ_1, due primarily to the microhydrodynamic resistances acting. Yet there is no consideration that τ might also be a function of r_{up} or φ_1. Nor is it generally considered explicitly that induction time may vary stochastically.

In this work we investigate the theoretical and experimental evidence for variation of the induction period for particle–bubble attachment. Only by completing our understanding of induction time can a rational comparison with sliding time be allowed, and thereby a prediction of attachment.

2. Motivation

Our exploration of possible variation in the induction period as a function of r_{up} or φ_1 is motivated by two main factors: recognition of variation in an uncontrolled parameter; and a desire to ensure physical meaningfulness. These are expanded upon below. Each of the foregoing factors also bears upon the issue of comparing estimates of induction period obtained from different techniques, which provides a further rationale for the work.

2.1. Accounting for variation in an uncontrolled parameter

Our primary interest is in estimating induction periods for different systems, in order to establish the effect of, say, different collectors, different pH, or different particle size. Of course, the preferred experimental approach is to vary as few parameters as possible, in order to establish causation, and to propose functional correlations. Ideally only one parameter would be varied at a time.

In our experimental observations of particle–bubble interaction it has not been found feasible to control to micrometre precision the upstream approach trajectory of individual particles. As will be described below, a dilute swarm of particles descends, covering a broad range of r_{up} and φ_1. Hence, r_{up} (or φ_1) represents an uncontrolled parameter that varies significantly. The importance of this depends upon whether or not the induction period is sensitive to that variation.

Although it was not feasible to control φ_1 or r_{up}, values of these parameters can readily be estimated by direct observation. Fig. 2 illustrates schematically how these estimates might assist the comparison of induction period data for two different hypothetical systems, "A" and "B".

In Fig. 2a the induction time estimates are shown for the two systems. There is a large degree of variation in τ for each system, which is much larger than the difference between the respective averages. The induction periods for system A and system B are spread over a similar range. Without any further information, we

[2] "Sliding" strictly applies only to unattached particles — hence Δt_{slide} represents a maximum duration available for attachment to occur. The motion after attachment occurs should be distinguished from sliding, which we do by referring to it as "gliding". Our observations (Verrelli et al., 2011) suggest that in practice the motion of sliding and gliding particles will often be similar, although that is not required in the present analysis.

[3] To reduce the computational load where many particles and bubbles are simulated, the efficiencies can be expressed as a function of φ_1 or r_{up} and then integrated to obtain a suitably weighted average; this average efficiency is then stored and used to describe all encounters in a probabilistic fashion (*cf.* Yoon and Luttrell, 1989; Koh and Schwarz, 2006).